Technological cooperation between countries against a terrorist threat

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Abstract

The aim of the paper is to analyze the cooperation between countries faced to a terrorist threat through a cooperative-game-theoretic approach. The countries are differentiated by their technology level in term of protection. Each government decides the part of its GDP allowed to the protection. The effectiveness of the protection is the result of the combination between its part of the GDP and the technology level. It will have an impact on the terrorist attack probability and on the maximum damage that a terrorist group is able to do. In order to counter this attack, countries have the possibility to cooperate, i.e. to form an alliance. The impact of such alliance is to improve the protection by a technology transfer.

Keywords: terrorist threat, cooperative games, Core, Shapley Value.

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1 Introduction

Nowadays the terrorist phenomenon is omnipresent. This fear has increased in western countries since the 11^{th} September attacks. However, for some countries, this fear is present every day. This is the case of the Palestine and Israel where the threat is permanent. Indeed, since 1947 and 1948, when the United Nations approved the partition plan of the Palestine and when Israel has had its independence, these two countries continue to attack each other through suicide attacks. These confrontations have done lots of destructions and will do it again in the future in spite of peace treaties, which have failed.

Some economists have studied the alliance of countries faced to a conflict. Lee (1988) focuses on the phenomenon of free rider in the case of alliances. Sorokin (1994) studies the security of a country through the choice of having more arms, forming an alliance, or a mix of these. Sandler (1999) analyzes a model of alliance formation to counter a threat along the borders. Moreover La Manna (1993), Ferrer and Valognes (1999) have studied the technology transfer in the case of firms.

Faced to terrorist threats, alliances have been formed. We can take the example of the NATO. It has been created in 1949. Its aim is to guarantee the security of its members. In 1994, the Partnership for Peace (PfP) was established in order to attenuate threats, to increase the stability, and to establish toughened relations between partners. The Prague Summit took place in 2002. There was the creation of a Partnership Action Plan on Terrorism which improves the border security, the cooperation and the sharing of information. The 2004 Istanbul Summit leads to a work programme in order to develop new advanced technologies. For example, the France leads works in the detection, the protection and the defeat of chemical, biological, radiological and nuclear weapons.

The members of an alliance benefit from these new technologies. However, in which cases is it profitable to transfer its technology to other countries ? The purpose of the paper is to analyze it in the case of two countries, then three, where there is only one country which has an advanced technology in terms of protection.

2 The Model

2.1 Hypotheses

We consider a set of n countries which are faced to a terrorist threat. Countries are located in the same geographic zone. In order to counter this threat or to attenuate this one, each country decides to protect itself. We suppose that they can not attack each other. The terrorist threat is represented by one terrorist group. This one can attack only once, i.e. one country during a period. However, an attack probability is attributed to all countries.

Among the set of countries, only one has a technology which improves its protection. The technology mixed with the protection of the country amplifies the effectiveness of the protection. Technology costs are equal to 0 because we suppose that this technology is initial to the country.

To fight against this threat, some countries will choose or not to cooperate. By forming an alliance, it will improve the protection. This improvement can be justified by the sharing of information on the terrorists. In this case, it is easier to counter them. The cooperation between the country having technology and an other one leads to a technological transfer to this last one. We consider that this transfer costs are equal to 0 because the knowledge is easily transferable.

The protection cost C_i is a GDP part of the country i :

$$C_i = c_i Y_i$$

where c_i $(0 < c_i < 1)$ is the GDP percentage allowed to the protection and Y_i the GDP. We supposed that the leading country (the country which has the technology) can improve its protection thanks to its technology. In this case, the total protection will be designated by T_i, where $T_i = \gamma C_i$, with γ the technological factor. γ is supposed to be exogenous.

The terrorist threat M_i, described by the following equation, represents the difference between the maximum terrorist damage and the protection of the country i (toughened or not by the technology).

$$M_i = m - max\{C_i, T_i\}$$

The power of the terrorist group is symbolized by the factor m. Indeed, the terrorist group is weak if the maximum damage m are lower than the protection effectiveness. To the contrary, it will be supposed strong if it manages to wreak hover on a country in spite of the protection. In the first case, terrorists have no interest to attack. In this model, we keep the following assumption : $\begin{cases} m > C_i \geq 0 \\ m > T_i \\ m < Y_i \\ M_i \geq 0 \\ \alpha_i \geq 0 \end{cases}$

The damage from a terrorist attack can't be higher than the GDP of a country i.

Remark 1. *The protection costs, the technology, the maximum damage and the terrorist threat are in monetary units. In this case the maximum damage represents the maximal amount of damage. The terrorist threat is the real amount of material loss. The technology combined with the protection is the amount of material saving if the country is attacked.*

We have defined the terrorist threat for the countries. But, if the group attacks, the target will be only one country. Each country has a probability to be attacked. This probability is designated by α_i:

$$\alpha_i = \frac{\frac{m}{n} - max\{C_i, T_i\} + \frac{1}{n-1}\sum_{i \neq j} max\{C_j, T_j\}}{m}$$

The factor α_i depends on the probability of the country i and the protection of other countries. The numerator is composed of the average threat per country which we deduct the protection of the country i, then we add the average protection of other countries. In the denominator we have the total terrorist threat.

This probability for the country i decreases when the protection of this country increases (even if it has the technology or not). To the contrary, it increases as one goes along the protection of other countries grows . Indeed, it transfers the risk to the other country. The goal of the terrorist group will be to attack the country where the damage is the highest [1].

Remark 2. *The sum of the probabilities of being attacked is lower or equal to 1 :* $\sum \alpha_i \leq 1$

We obtain the expected welfare function B_i for the country i:

$$B_i = Y_i - C_i - \alpha_i M_i, \quad \forall\ i = 1, ..., n$$

$$B_i = Y_i - C_i - \frac{\frac{m}{n} - max\{C_i, T_i\} + \frac{1}{n-1}\sum_{i \neq j} max\{C_j, T_j\}}{m}(m - max\{C_i, T_i\})$$

From this function each country maximizes its welfare. However before formulating this game under a characteristic form we present some definitions and concepts of cooperative games.

[1] See appendix 1

2.2 Concepts and definitions of cooperative games

A game under a characteristic form, composed of N players and a characteristic function v, which assigns the maximal payoff to each possible coalition, describes the cooperative possibilities. It is assumed that the empty coalition has no value: $v(\emptyset) = 0$

Definition 1. *A game is superadditive if for any disjoint coalitions $S, T \subseteq N$ we have :*

$$if\ \ S \cap T = \emptyset, \quad v(S \cup T) \geq v(S) + v(T)$$

Definition 2. *A game is convex if for all $S, T \subseteq N$ we have :*

$$v(S \cup \{i\}) - v(S) \leq v(T \cup \{i\}) - v(T), \qquad \forall\ i \in N \ \ and\ \ \forall\ S \subseteq T \subseteq N - \{i\}$$

The two well-known solution concepts in cooperative games are the Core and the Shapley value. The Core may be empty or quite large, whereas the Shapley value defines a single solution.

Definition 3. *The Core of a game* v *is the set of feasible utility outcomes with the property that no coalition could on its own improve the payoffs of all its members. For all $S \subset N$ we have:*

$$(i) \quad \sum_{i \in S} x_i \geq v(S)$$

$$(ii) \quad \sum_{i \in N} x_i = v(N)$$

Definition 4. *The Shapley value of a cooperative game is denoted by :*

$$\varphi_i(v) = \sum_{S \subset N, i \in S} \frac{(s-1)!(n-s)!}{n!}[v(S) - v(S - \{i\})], \ with\ s = |S|\ and\ n = |N|.$$

$\varphi_i(v)$ is the average marginal contribution of i *to all the possible coalitions.*

3 Case of two countries

3.1 Characteristic functions

We consider two countries or two groups of representative countries P_1 et P_2. We suppose that one of these countries has an advanced technology in terms of protection. We define P_1 as the leading country. This country has the following welfare function :

$$B_1 = Y_1 - C_1 - \alpha_1 M_1, \text{ with } \begin{cases} \alpha_1 = \frac{\frac{m}{2} - T_1 + C_2}{m} \\ M_1 = m - T_1 \\ T_1 = \gamma C_1 \end{cases}$$

For the other country, its welfare function is defined by :

$$B_2 = Y_2 - C_2 - \alpha_2 M_2, \text{ with } \begin{cases} \alpha_2 = \frac{\frac{m}{2} - C_2 + T_1}{m} \\ M_2 = m - C_2 \\ T_1 = \gamma C_1 \\ T_2 = C_1 \end{cases}$$

Each country wants to maximize its welfare considering its protection. So we have :

$$\max_{C_i > 0} B_i = Y_i - C_i - \alpha_i M_i$$

Proposition 1. *In the situation where there are no coalitions, we obtain :*

$$\begin{cases} \hat{C}_1 = \frac{m(7\gamma-4)}{6\gamma^2} \\ \hat{C}_2 = \frac{m(5\gamma-2)}{6\gamma} \end{cases} \begin{cases} \hat{\alpha}_1 = \frac{\gamma+2}{6\gamma} \\ \hat{\alpha}_2 = \frac{5\gamma-2}{6\gamma} \end{cases} \begin{cases} \hat{M}_1 = \frac{m(4-\gamma)}{6\gamma} \\ \hat{M}_2 = \frac{m(\gamma+2)}{6\gamma} \end{cases} \begin{cases} \hat{B}_1 = Y_1 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \\ \hat{B}_2 = Y_2 + \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \end{cases}$$

Proof : See appendix 2.

Remark 3. *From $\gamma \geq 4$, the threat to the leading country is not credible. From this value, we suppose that the threat is equal to 0. In this case the country 1 is outside the game. Only the country 2 is threatened. We have : $B_2 = Y_2 - C_2 - \alpha_2 M_2$ with $\alpha_2 = \frac{m-C_2}{m}$ and $M_2 = m - C_2$ After maximizing its welfare, we obtain :*

$\hat{C}_2 = \frac{m}{2}$

$\hat{\alpha}_2 = \frac{1}{2}$

$\hat{M}_2 = \frac{m}{2}$

$\hat{B}_2 = Y_2 - \frac{3m}{4}$

Remark 4. *If the technological factor is equal to 1 (i.e. the leading country has not this technology or the two countries have the same technology level), so the protection costs, the probability of being attacked and the terrorist threat are the same for them. The welfare function can be different because the countries can have different GDP.*

The protection costs increases with the terrorist threat. Indeed, the more a terrorist group is strong, the more the protection costs should be high in order to ease this threat. The damage to the country 1 increase with the power of a terrorist group for a low technology level (for $\gamma < 4$). To the contrary, for a high technological level, the damage decrease with the terrorist power because the group decides not to attack for a certain technology level (This is the reason why the damage decrease with this level). For the country 2, the damage grow with the terrorist strength and diminish with the technological factor of the leading country. The factor γ conduces the country 2 to rise its protection to avoid the risk transfer on it. By increasing its protection spending, it results on a lowering of the terrorist damage.

The probability of being attacked is decreasing with the technological factor for the country 1. On the contrary it is growing for the other country. The probability is increasing for the country 2 because the risk will be transferred on it given that the country 1 is best-protected[2].

Proposition 2. *Globally, the welfare of the two countries decreases with the strength of a terrorist group, except for the leading country thanks to its technology for a certain level ($\gamma > 43,6$) because its protection will deter terrorists.*

Proof : See appendix 4

We suppose that the two countries want to cooperate. In this case, the country 2 benefits from the advanced technology of the leading country. This alliance has a common protection cost. They maximize their welfare with respect to the protection costs considering the terrorist threat. We define B_{12}, C_{12}, α_{12}, M_{12}, respectively as the common welfare function, the common protection costs, the common probability of being attacked and the common terrorist damage for the two countries.

$$\max_{C_{12}>0} B_{12} = Y_{12} - C_{12} - \alpha_{12}M_{12}, \text{ with } \begin{cases} Y_{12} = Y_1 + Y_2 \\ \alpha_{12} = \frac{m-\gamma C_{12}}{m} \\ M_{12} = m - \gamma C_{12} \end{cases}$$

Proposition 3. *We have :*
$\hat{C}_{12} = \frac{m(2\gamma-1)}{2\gamma^2}$
$\hat{\alpha}_{12} = \frac{1}{2\gamma}$
$\hat{M}_{12} = \frac{m}{2\gamma}$
$\hat{B}_{12} = Y_1 + Y_2 + \frac{m(1-4\gamma)}{4\gamma^2}$

Proof : See appendix 5

As for the case of no-coalition, the protection costs increases with the terrorist strength and decreases with the factor α. The technological factor has positive repercussion for the countries because it will decrease the probability of attack and the terrorist damage, and it will increase the common welfare. The factor m has contrary effects to the factor γ.

Proof : See appendix 6

Remark 5. *In the case of no-coalition, the damage* M *are decreasing with the terrorist power for a certain level. Indeed, the group will not attack the strongest country but the other one. In the case of a coalition, the both countries are the only target for the terrorist, so the risk will be not transferred somewhere else.*

[2]See Appendix 3

The expected welfare functions define the characteristic functions of this game :

$$\begin{cases} v(1) = Y_1 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \\ v(2) = Y_2 + \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \\ v(12) = Y_1 + Y_2 + \frac{m(1-4\gamma)}{4\gamma^2} \end{cases}$$

The characteristic functions of the game are now defined. Is a coalition optimal in the case of two countries ? The core and the Shapley value provide us an answer.

3.2 Results

The well-known solutions to cooperative games is the Core and the Shapley value. Thanks to these concepts we could determine the coalition.

Theorem 1. *(Shapley, 1971) The Core of a convex game is non-empty.*

Proposition 4. *The core of this game is not empty because this game is convex.*

Proof : See appendix 7

An allocation $x = \{x_1, x_2\}$ is in the core if and only if :

$$\begin{cases} x_1 \qquad \geq Y_1 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \\ \qquad x_2 \geq Y_2 + \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \\ x_1 + x_2 = Y_1 + Y_2 + \frac{m(1-4\gamma)}{4\gamma^2} \end{cases}$$

Proposition 5. *The Core of the game is :*
$C(v) = \{(x_1; Y_1 + Y_2 + \frac{m(1-4\gamma)}{4\gamma^2} - x_1) | Y_1 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \leq x_1 \leq Y_1 + \frac{m(35\gamma^2-40\gamma+5)}{36\gamma^2}\}$

All these allocations define the core for which the grand coalition is preferable, because the welfare of the countries is higher than this one when they don't cooperate.

We have defined the core of this game. Now we specify the Shapley value. The advantage of this value is that it give us a single solution.

Theorem 2. *(Shapley, 1971) The Shapley value of a convex game belongs to the Core of this game.*

Proposition 6. *The Shapley value $\varphi = (\varphi_1, \varphi_2)$, belonging to the core, has the following solution :*

$$\begin{cases} \varphi_1 = Y_1 + \frac{m(12\gamma^2-28\gamma+7)}{24\gamma^2} \\ \varphi_2 = Y_2 + \frac{m(-12\gamma^2+4\gamma-1)}{24\gamma^2} \end{cases}$$

In the case of two countries, the optimal solution is to cooperate if the technology level of the leading country is low ($\gamma < 4$). On the contrary, if the technology level is high, the leading country prefers staying alone. Indeed the threat is not credible. This country is too well protected. The terrorist group wants to attack the other country.

4 Case of three countries

Now we consider the case of three countries. There are more possibilities of alliances. We take back the hypotheses. The country 1 is the only country which has an advanced technology. It is the leading country. The others differentiated by their GDP don't have technology because they are weak. Therefore :

$$B_i = Y_i - C_i - \alpha_i M_i, \ \forall\, i = 1, 2, 3 \text{ with } \begin{cases} \alpha_i = \frac{\frac{m}{3} - max\{C_i, T_i\} + \frac{1}{2}\sum_{i \neq j} max\{C_j, T_j\}}{m} \\ M_i = m - max\{C_i, T_i\} \end{cases}$$

Before the maximization of the welfare for each country or coalition, it is necessary to consider several cases. Indeed if the country i decides not to form an alliance, then the other countries have the possibilities to cooperate or not. So, as Rajan [1989], we distinguish different games :

- First, the case where there are no complementary coalitions (Game G_1). When a coalition is formed, the other countries choose to stay alone.
- Second, the game G_2 describes the situation when the complementary coalition is formed. When a coalition is formed, the other countries choose to form the complementary coalition.

4.1 The game G_1

4.1.1 Characteristic functions

This case means that if a country i decides to stay alone, the other countries will not form the complementary coalition. The value of a coalition S is given by : $\begin{cases} \max_{C_S \geqslant 0} B_S(C_S, \hat{C}_{N-S}) \\ \max_{C_j \geqslant 0} B_j(\hat{C}_S, C_j, \hat{C}_{N-S-\{j\}}) \end{cases}$

As previously we maximize the welfare function with respect to protection costs for each country.

Each country decides to allocate the following amount to the protection. We obtain the probability of an attack, the amount of damage and the expected welfare function in the case of no coalition:

$$\begin{cases} \hat{C}_1^{(1)(2)(3)} = \frac{m(14\gamma-9)}{15\gamma^2} \\ \hat{C}_2^{(1)(2)(3)} = \frac{m(8\gamma-3)}{15\gamma} \\ \hat{C}_3^{(1)(2)(3)} = \frac{m(8\gamma-3)}{15\gamma} \end{cases} \begin{cases} \hat{\alpha}_1^{(1)(2)(3)} = \frac{6-\gamma}{15\gamma} \\ \hat{\alpha}_2^{(1)(2)(3)} = \frac{8\gamma-3}{15\gamma} \\ \hat{\alpha}_3^{(1)(2)(3)} = \frac{8\gamma-3}{15\gamma} \end{cases} \begin{cases} \hat{M}_1^{(1)(2)(3)} = \frac{m(\gamma+9)}{15\gamma} \\ \hat{M}_2^{(1)(2)(3)} = \frac{m(7\gamma+3)}{15\gamma} \\ \hat{M}_3^{(1)(2)(3)} = \frac{m(7\gamma+3)}{15\gamma} \end{cases}$$

$$\begin{cases} \hat{B}_1^{(1)(2)(3)} = Y_1 + \frac{m(\gamma^2-207\gamma+81)}{225\gamma^2} \\ \hat{B}_2^{(1)(2)(3)} = Y_2 + \frac{m(-176\gamma^2+42\gamma+9)}{225\gamma^2} \\ \hat{B}_3^{(1)(2)(3)} = Y_3 + \frac{m(-176\gamma^2+42\gamma+9)}{225\gamma^2} \end{cases}$$

Remark 6. *The influence of the parameters on these functions for the case with two countries are the same for the case with three countries but the values for γ are different.*

There are many possibilities of coalition. The leading country can join the coalition and by the fact, transferring its technology to it(s) partner(s) :

$$\max_{C_{1j}>0} B_{1j} = Y_{1j} - C_{1j} - \alpha_{1j} M_{1j}, \forall\ j = 2,3 \text{ with } \begin{cases} Y_{1j} = Y_1 + Y_j \\ \alpha_{1j} = \frac{m-\gamma C_{1j}}{m} \\ M_{1j} = m - \gamma C_{1j} \end{cases}$$

The amount of protection, the probabilities of attack, the threat and the expected welfare for the coalition {1,j} will be :

$$\begin{cases} \hat{C}_{1j}^{(1j)(k)} = \frac{m(7\gamma-4)}{6\gamma^2} \\ \hat{C}_k^{(1j)(k)} = \frac{m(5\gamma-2)}{6\gamma} \end{cases} \begin{cases} \hat{\alpha}_{1j}^{(1j)(k)} = \frac{\gamma+2}{6\gamma} \\ \hat{\alpha}_k^{(1j)(k)} = \frac{5\gamma-2}{6\gamma} \end{cases} \begin{cases} \hat{M}_{1j}^{(1j)(k)} = \frac{m(4-\gamma)}{6\gamma} \\ \hat{M}_k^{(1j)(k)} = \frac{m(\gamma+2)}{6\gamma} \end{cases}$$

$$\begin{cases} \hat{B}_{1j}^{(1j)(k)} = Y_1 + Y_j + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \\ \hat{B}_k^{(1j)(k)} = Y_k + \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \end{cases}$$

Inside the coalition {2,3} there is no technology transfer. As previously this coalition wants to maximize its welfare :

$$\max_{C_{23}>0} B_{23} = Y_{23} - C_{23} - \alpha_{23} M_{23}, \text{ with } \begin{cases} Y_{23} = Y_2 + Y_3 \\ \alpha_{23} = \frac{m-\gamma C_{23}}{m} \\ M_{23} = m - C_{23} \end{cases}$$

The amount of protection, the probabilities of attack, the threat and the expected welfare for the coalition {2,3} will be :

$$\begin{cases} \hat{C}_{23}^{(23)(1)} = \frac{m(5\gamma-2)}{6\gamma} \\ \hat{C}_1^{(23)(1)} = \frac{m(7\gamma-4)}{6\gamma^2} \end{cases} \begin{cases} \hat{\alpha}_{23}^{(23)(1)} = \frac{5\gamma-2}{6\gamma} \\ \hat{\alpha}_1^{(23)(1)} = \frac{\gamma+2}{6\gamma} \end{cases} \begin{cases} \hat{M}_{23}^{(23)(1)} = \frac{m(\gamma+2)}{6\gamma} \\ \hat{M}_1^{(23)(1)} = \frac{m(4-\gamma)}{6\gamma} \end{cases}$$

$$\begin{cases} \hat{B}_{23}^{(23)(1)} = Y_2 + Y_3 \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \\ \hat{B}_1^{(23)(1)} = Y_1 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \end{cases}$$

Now if the countries chooses to form an alliance, the technology will be available for all countries. Therefore :

$\hat{C}_{123}^{(123)} = \frac{m(2\gamma-1)}{2\gamma^2}$

$\hat{\alpha}_{123}^{(123)} = \frac{1}{2\gamma}$

$\hat{M}_{123}^{(123)} = \frac{m}{2\gamma}$
$\hat{B}_{123}^{(123)} = Y_1 + Y_2 + Y_3 + \frac{m(-4\gamma+1)}{4\gamma^2}$

The expected welfare functions define the characteristic functions of this game :

$$\begin{cases} v_1^{(1)(2)(3)} = Y_1 + \frac{m(\gamma^2-207\gamma+81)}{225\gamma^2} \\ v_2^{(1)(2)(3)} = Y_2 + \frac{m(-176\gamma^2+42\gamma+9)}{225\gamma^2} \\ v_3^{(1)(2)(3)} = Y_3 + \frac{m(-176\gamma^2+42\gamma+9)}{225\gamma^2} \\ v_{12}^{(12)(3)} = Y_1 + Y_2 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \\ v_{13}^{(13)(2)} = Y_1 + Y_3 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \\ v_{23}^{(23)(1)} = Y_2 + Y_3 + \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \\ v_{123}^{(123)} = Y_1 + Y_2 + Y_3 + \frac{m(-4\gamma+1)}{4\gamma^2} \end{cases}$$

We have defined the characteristic functions in order to know which coalition will be formed and consequently to know if a technology transfer will be possible.

4.1.2 Results

To know which coalition is preferable for everyone, the concept of the Core is the most appropriate.

An allocation $x = \{x_1, x_2, x_3\}$ is in the core if and only if :

$$\begin{cases} x_1 \geq Y_1 + \frac{m(\gamma^2-207\gamma+81)}{225\gamma^2} \\ x_2 \geq Y_2 + \frac{m(-176\gamma^2+42\gamma+9)}{225\gamma^2} \\ x_3 \geq Y_3 + \frac{m(-176\gamma^2+42\gamma+9)}{225\gamma^2} \\ x_1 + x_2 \geq Y_1 + Y_2 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \\ x_1 + x_3 \geq Y_1 + Y_3 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \\ x_2 + x_3 \geq Y_2 + Y_3 + \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \\ x_1 + x_2 + x_3 = Y_1 + Y_2 + Y_3 + \frac{m(-4\gamma+1)}{4\gamma^2} \end{cases}$$

Proposition 7. *The Core of the game G_1 is :*
$C(v) = \{(x_1, x_2, x_3\}|Y_1 + \frac{m(\gamma^2-207\gamma+81))}{225\gamma^2} \leq x_1 \leq Y_1 + \frac{m(35\gamma^2-40\gamma+5)}{36\gamma^2}, Y_2 + \frac{m(-176\gamma^2+42\gamma+9)}{225\gamma^2} \leq x_2 \leq Y_2 + \frac{m(-\gamma^2+8\gamma-7)}{36\gamma^2}, Y_3 + \frac{m(-176\gamma^2+42\gamma+9)}{225\gamma^2} \leq x_3 \leq Y_3 + \frac{m(-\gamma^2+8\gamma-7)}{36\gamma^2}, x_1 + x_2 + x_3 = Y_1 + Y_2 + Y_3 + \frac{m(-4\gamma+1)}{4\gamma^2}\}$

Proposition 8. *The Core is not empty because the game is convex for a certain value of $\gamma : \gamma \in]1; 9.7]$. Beyond this value the grand coalition is not optimal for some countries.*

Proof : See Appendix 8

Proposition 9. *The Shapley value* $\varphi = (\varphi_1, \varphi_2, \varphi_3)$ *belonging to the Core, has the following solution:*

$$\begin{cases} \varphi_1 = Y_1 + \frac{m(536\gamma^2 - 1032\gamma + 271)}{900\gamma^2} \\ \varphi_2 = Y_2 + \frac{m(-268\gamma^2 + 66\gamma - 23)}{900\gamma^2} \\ \varphi_3 = Y_3 + \frac{m(-268\gamma^2 + 66\gamma - 23)}{900\gamma^2} \end{cases}$$

In the case of three countries, the conclusions are different than in the case of two countries. When the technology of the leading country is very low ($1 < \gamma < 4$), in this case the cooperation (i.e. the grand coalition) will be the best solution. When the technology of the leading country is low ($4 < \gamma < 6$), the leading country will form an alliance with one of the two countries. The terrorist attack will be equal to 0. However the leading country refuses to join the grand coalition from a certain level of technology ($\gamma > 6$). The attack probability on the leading country is equal to 0 in this case. So the threat is not credible. The two other countries will form a coalition.

4.2 The game G_2

4.2.1 Characteristic functions

In this game we consider that if a coalition is formed, then the other countries form the complementary coalition. The value of the coalition S is given by:

$$\begin{cases} \max_{C_S \geqslant 0} B_S(C_S, \hat{C}_{N-S}) \\ \max_{C_j \geqslant 0} B_j(\hat{C}_S, C_{N-S}) \end{cases}$$

For example if the country 1 decides to stay alone, the coalition {2,3} will be formed.

The characteristic functions have been calculated in the previous part. Therefore, the game G_2 is defined by the following functions :

$$\begin{cases} v_1^{(1)(23)} = Y_1 + \frac{m(\gamma^2 - 44\gamma + 16)}{36\gamma^2} \\ v_2^{(13)(2)} = Y_2 + \frac{m(-35\gamma^2 + 4\gamma + 4)}{36\gamma^2} \\ v_3^{(12)(3)} = Y_3 + \frac{m(-35\gamma^2 + 4\gamma + 4)}{36\gamma^2} \\ v_{12}^{(12)(3)} = Y_1 + Y_2 + \frac{m(\gamma^2 - 44\gamma + 16)}{36\gamma^2} \\ v_{13}^{(13)(2)} = Y_1 + Y_3 + \frac{m(\gamma^2 - 44\gamma + 16)}{36\gamma^2} \\ v_{23}^{(23)(1)} = Y_2 + Y_3 + \frac{m(-35\gamma^2 + 4\gamma + 4)}{36\gamma^2} \\ v_{123}^{(123)} = Y_1 + Y_2 + Y_3 + \frac{m(-4\gamma + 1)}{4\gamma^2} \end{cases}$$

4.2.2 Results

An allocation $x = \{x_1, x_2, x_3\}$ is in the core if and only if :

$$\begin{cases} x_1 \geq Y_1 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \\ x_2 \geq Y_2 + \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \\ x_3 \geq Y_3 + \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \\ x_1 + x_2 \geq Y_1 + Y_2 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \\ x_1 + x_3 \geq Y_1 + Y_3 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \\ x_2 + x_3 \geq Y_2 + Y_3 + \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \\ x_1 + x_2 + x_3 = Y_1 + Y_2 + Y_3 + \frac{m(-4\gamma+1)}{4\gamma^2} \end{cases}$$

Proposition 10. *The Core of the game G_2 is :*
$C(v) = \{(x_1, x_2, x_3)|Y_1 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \leq x_1 \leq Y_1 + \frac{m(35\gamma^2-40\gamma+5)}{36\gamma^2}, Y_2 + \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \leq x_2 \leq Y_2 + \frac{m(-\gamma^2+8\gamma-7)}{36\gamma^2}, Y_3 + \frac{m(-35\gamma^2+4\gamma+4)}{36\gamma^2} \leq x_3 \leq Y_3 + \frac{m(-\gamma^2+8\gamma-7)}{36\gamma^2}, x_1 + x_2 + x_3 = Y_1 + Y_2 + Y_3 + \frac{m(-4\gamma+1)}{4\gamma^2}\}$

Proposition 11. *The Core is not empty because the game is convex for a certain value of γ : $\gamma \in]1; 7]$.*

Proof : See Appendix 9

Proposition 12. *The Shapley value $\varphi = (\varphi_1, \varphi_2, \varphi_3)$ belonging to the Core, has the following solution:*

$$\begin{cases} \varphi_1 = Y_1 + \frac{m(24\gamma^2-44\gamma+11)}{36\gamma^2} \\ \varphi_2 = Y_2 + \frac{m(-12\gamma^2+4\gamma-1)}{36\gamma^2} \\ \varphi_3 = Y_3 + \frac{m(-12\gamma^2+4\gamma-1)}{36\gamma^2} \end{cases}$$

In this game we have the same results as the previous game.

5 Conclusion

This paper develops a model in which some countries are faced to a terrorist threat. We wonder if the cooperation is the best solution. If there is a cooperation, the technology in terms of protection will be transferred to the partners of the alliance. Our main result is that a low technology level of the leading country shows that this one is a potential target. So the coalition with all countries is profitable for all.
From a certain technology level, an alliance between the leading country and an other one will be sufficient to avoid a terrorist threat. The risk of an attack is transferred to the country staying alone.
In the case where the leading country is well protected thanks to its technology, it prefers keeping it and staying alone. The threat on it will not be credible in this case.
The results are checked in the case of two and three countries.

However we could generalize to n countries and consider that these countries have a technology too (but the technology level will be different for each country). Moreover the possibility of an alliance between a country and a terrorist group is conceivable. In this case the terrorist group will be considered as a player.

6 Appendix

6.1 appendix 1

$\frac{\delta \alpha_i}{\delta m} = 0$

$\frac{\delta \alpha_i}{\delta C_i} = \frac{-1}{m} < 0$

$\frac{\delta \alpha_i}{\delta T_i} = \frac{-1}{m} < 0$

$\frac{\delta \alpha_i}{\delta C_j} = \frac{1}{m} > 0$

$\frac{\delta \alpha_i}{\delta T_j} = \frac{1}{m} > 0$

6.2 appendix 2

In the case of no coalition, each country maximizes its welfare with respect to its protection costs.
$\max_{C_i > 0} B_i = Y_i - C_i - \alpha_i M_i$

For the country 1 :
$\max_{C_1 > 0} B_1 = Y_1 - C_1 - \frac{\frac{m}{2} - \gamma C_1 + C_2}{m}(m - \gamma C_1)$
$\frac{\delta B_1}{\delta C_1} = 0$
$C_1 = \frac{2\gamma C_2 + m(3\gamma - 2)}{4\gamma^2}$

For the country 2 :
$\max_{C_2 > 0} B_2 = Y_2 - C_2 - \frac{\frac{m}{2} - C_2 + \gamma C_1}{m}(m - C_2)$
$\frac{\delta B_2}{\delta C_2} = 0$
$C_2 = \frac{m}{4} + \frac{\gamma C_1}{2}$

After solving the system, we obtain :
$$\begin{cases} \hat{C}_1 = \frac{m(7\gamma - 4)}{6\gamma^2} \\ \hat{C}_2 = \frac{m(5\gamma - 2)}{6\gamma} \end{cases}$$
We substitute the $\hat{C}_i$ in order to obtain the probability of an terrorist attack, the maximum damage and the welfare functions :
$$\begin{cases} \hat{\alpha}_1 = \frac{\gamma + 2}{6\gamma} \\ \hat{\alpha}_2 = \frac{5\gamma - 2}{6\gamma} \end{cases} \begin{cases} \hat{M}_1 = \frac{m(4 - \gamma)}{6\gamma} \\ \hat{M}_2 = \frac{m(\gamma + 2)}{6\gamma} \end{cases} \begin{cases} \hat{B}_1 = Y_1 + \frac{m(\gamma^2 - 44\gamma + 16)}{36\gamma^2} \\ \hat{B}_2 = Y_2 + \frac{m(-35\gamma^2 + 4\gamma + 4)}{36\gamma^2} \end{cases}$$

6.3 appendix 3

Influence of terrorist power and the technology of the country 1 on the protection costs :
$\frac{\delta \hat{C}_1}{\delta m} = \frac{7\gamma-4}{6\gamma^2} > 0$, because $\gamma > 1$

$\frac{\delta \hat{C}_2}{\delta m} = \frac{5\gamma-2}{6\gamma} > 0$

$\frac{\delta \hat{C}_1}{\delta \gamma} = \frac{-m(7\gamma-8)}{6\gamma^3}$

$$\begin{cases} \frac{\delta \hat{C}_1}{\delta \gamma} > 0 \ \ for \ \ \gamma < \frac{8}{7} \\ \frac{\delta \hat{C}_1}{\delta \gamma} < 0 \ \ otherwise \end{cases}$$

$\frac{\delta \hat{C}_2}{\delta \gamma} = \frac{m}{3\gamma^2} > 0$

Influence of terrorist power and the technology of the country 1 on the damage :
$\frac{\delta \hat{M}_1}{\delta m} = \frac{4-\gamma}{6\gamma}$

$$\begin{cases} \frac{\delta \hat{M}_1}{\delta m} < 0 \ \ for \ \ \gamma < 4 \\ \frac{\delta \hat{M}_1}{\delta m} > 0 \ \ otherwise \end{cases}$$

$\frac{\delta \hat{M}_2}{\delta m} = \frac{\gamma+2}{6\gamma} > 0$

$\frac{\delta \hat{M}_1}{\delta \gamma} = \frac{-2m}{3\gamma^2} < 0$

$\frac{\delta \hat{M}_2}{\delta \gamma} = \frac{-m}{3\gamma^2} < 0$

Infuence of technology on the probability to be attacked :
$\frac{\delta \hat{\alpha}_1}{\delta \gamma} = \frac{-1}{3\gamma^2} < 0$

$\frac{\delta \hat{\alpha}_2}{\delta \gamma} = \frac{1}{3\gamma^2} > 0$

Influence of the terrorist power and the technology on the welfare :

$$\frac{\delta \hat{B}_1}{\delta m} = \frac{\gamma^2-44\gamma+16}{36\gamma^2} \begin{cases} < 0 \ \ for \ \ 1 < \gamma < 22 + 6\sqrt{13} \simeq 43.6 \\ > 0 \ \ otherwise \end{cases}$$

$\frac{\delta \hat{B}_2}{\delta m} = \frac{-35\gamma^2+4\gamma+4}{36\gamma^2} < 0, \forall \gamma$

$\frac{\delta \hat{B}_1}{\delta \gamma} = \frac{m(11\gamma-8)}{9\gamma^3} > 0$

$\frac{\delta \hat{B}_2}{\delta \gamma} = \frac{m(\gamma+2)}{9\gamma^3} < 0$

6.4 appendix 4

$\hat{B}_1 = Y_1 + \frac{m(\gamma^2-44\gamma+16)}{36\gamma^2}$
Even if m is high, the protection will be always effectiveness if
$\frac{m(\gamma^2-44\gamma+16)}{36\gamma^2} \geq 0$

$\gamma^2 - 44\gamma + 16 \geq 0$ for $\gamma \geq 43.6$

6.5 appendix 5

In the case of a coalition, the common welfare function will be maximized about protection costs of the both countries. We have :
$\max_{C_{12}>0} B_{12} = Y_1 + Y_2 - C_{12} - \alpha_{12}M_{12}$ $\frac{\delta B_{12}}{\delta C_{12}} = 0$
$\hat{C}_{12} = \frac{m(2\gamma-1)}{2\gamma^2}$

We replace into the welfare functions, the probability and the threat function :

$\hat{C}_{12} = \frac{m(2\gamma-1)}{2\gamma^2}$
$\hat{\alpha}_{12} = \frac{1}{2\gamma}$
$\hat{M}_{12} = \frac{m}{2\gamma}$
$\hat{B}_{12} = Y_1 + Y_2 + \frac{m(1-4\gamma)}{4\gamma^2}$

6.6 appendix 6

Influence of parameters on the common protection costs :
$\frac{\delta\hat{C}_{12}}{\delta m} = \frac{2\gamma-1}{2\gamma^2} > 0$
$\frac{\delta\hat{C}_{12}}{\delta\gamma} = \frac{m(1-\gamma)}{\gamma^3} < 0$

Influence on the probability :
$\frac{\delta\hat{\alpha}_{12}}{\delta\gamma} = \frac{-1}{2\gamma^2} < 0$

Influence on the damage :
$\frac{\delta\hat{M}_{12}}{\delta m} = \frac{1}{2\gamma} > 0$
$\frac{\delta\hat{M}_{12}}{\delta\gamma} = \frac{-m}{2\gamma^2} < 0$

Influence on the welfare :
$\frac{\delta\hat{B}_{12}}{\delta m} = \frac{1-4\gamma}{4\gamma^2} < 0$

$\frac{\delta \hat{B}_{12}}{\delta \gamma} = \frac{m(2\gamma-1)}{2\gamma^3} > 0$

6.7 appendix 7

Convexity of the game :
A game is convex if :
$v(12) - v(1) - v(2) \geq 0$

$v(12) - v(1) - v(2) = \frac{m(34\gamma^2+4\gamma-11)}{36\gamma^2}$
$\frac{m(34\gamma^2+4\gamma-11)}{36\gamma^2} \geq 0$ for $\gamma \in [\frac{\sqrt{1512}-4}{68}; +\infty[$
$\gamma > 1$, so the game is convex and the core is not empty.

6.8 appendix 8

Conditions on convexity :
We have to show that :
$v(12) - v(1) - v(2) \geq 0$
$v(13) - v(1) - v(3) \geq 0$
$v(23) - v(2) - v(3) \geq 0$
$v(123) - v(12) - v(3) \geq 0$
$v(123) - v(23) - v(1) \geq 0$
$v(123) - v(13) - v(2) \geq 0$
$v(123) - v(23) - v(12) + v(2) \geq 0$
$v(123) - v(13) - v(23) + v(3) \geq 0$
$v(123) - v(12) - v(13) + v(1) \geq 0$
$2v(123) - v(12) - v(13) - v(23) \geq 0$

$v(12) - v(1) - v(2) \geq 0$
$\Leftrightarrow 145\gamma^2 - 88\gamma + 8 \geq 0$
As $\gamma > 1$, then $145\gamma^2 - 88\gamma + 8 \geq 0 > 0$
So $v(12) - v(1) - v(2) \geq 0$

$v(13) - v(1) - v(3) \geq 0$
$\Leftrightarrow 145\gamma^2 - 88\gamma + 8 \geq 0$
As $\gamma > 1$, then $145\gamma^2 - 88\gamma + 8 \geq 0 > 0$
$v(13) - v(1) - v(3) \geq 0$

$v(23) - v(2) - v(3) \geq 0$
$\Leftrightarrow 133.25\gamma^2 - 59\gamma + 7 \geq 0$
We have $\gamma > 1$ so $133.25\gamma^2 - 59\gamma > 0$
Therefore $v(23) - v(2) - v(3) \geq 0$

$v(123) - v(12) - v(3) \geq 0$

$v(123) - v(13) - v(2) \geq 0$
$\Leftrightarrow 679\gamma^2 + 32\gamma - 21 \geq 0$
As $\gamma > 1$, then $v(123) - v(1j) - v(k) \geq 0$, $\forall\ j = 2, 3$.

$v(123) - v(23) - v(1) \geq 0$
$\Leftrightarrow 217.75\gamma^2 - 43\gamma - 49.75 \geq 0$
As $\gamma > 1$, then it is positive.

$v(123) - v(23) - v(12) + v(2) \geq 0$
$v(123) - v(13) - v(23) + v(3) \geq 0$
$\Leftrightarrow 146\gamma^2 + 268\gamma - 239 \geq 0$
It is positive.

$v(123) - v(12) - v(13) + v(1) \geq 0$
$\Leftrightarrow -46\gamma^2 + 472\gamma - 251 \geq 0$
$v(123) - v(12) - v(13) + v(1) \geq 0$ for $\gamma < 9.7$

$2v(123) - v(12) - v(13) - v(23) \geq 0$
$\Leftrightarrow 11\gamma^2 + 4\gamma - 6 \geq 0$
It is positive.

6.9 appendix 9

Conditions on convexity :
We have to show that :
$v(12) - v(1) - v(2) \geq 0$
$v(13) - v(1) - v(3) \geq 0$
$v(23) - v(2) - v(3) \geq 0$
$v(123) - v(12) - v(3) \geq 0$
$v(123) - v(23) - v(1) \geq 0$
$v(123) - v(13) - v(2) \geq 0$
$v(123) - v(23) - v(12) + v(2) \geq 0$
$v(123) - v(13) - v(23) + v(3) \geq 0$
$v(123) - v(12) - v(13) + v(1) \geq 0$
$2v(123) - v(12) - v(13) - v(23) \geq 0$

$v(12) - v(1) - v(2) \geq 0$
$v(13) - v(1) - v(3) \geq 0$
$\Leftrightarrow 35\gamma^2 - 4\gamma - 4 \geq 0$
Positive

$v(23) - v(2) - v(3) \geq 0$
$\Leftrightarrow 35\gamma^2 - 4\gamma - 4 \geq 0$

It is positive.

$v(123) - v(12) - v(3) \geq 0$
$v(123) - v(13) - v(2) \geq 0$
$\Leftrightarrow 34\gamma^2 + 4\gamma - 11 \geq 0$
Positive

$v(123) - v(23) - v(1) \geq 0$
$\Leftrightarrow 34\gamma^2 + 4\gamma - 11 \geq 0$
$v(123) - v(23) - v(1) \geq 0$

$v(123) - v(23) - v(12) + v(2) \geq 0$
$v(123) - v(13) - v(23) + v(3) \geq 0$
$\Leftrightarrow -\gamma^2 + 8\gamma - 7 \geq 0$
Positive for $\gamma < 7$

$v(123) - v(12) - v(13) + v(1) \geq 0$
$\Leftrightarrow -\gamma^2 + 8\gamma - 7 \geq 0$
Positive for $\gamma < 7$

$2v(123) - v(12) - v(13) - v(23) \geq 0$
$\Leftrightarrow 11\gamma^2 + 4\gamma - 6 \geq 0$
Positive

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