# Inequality and a Repeated Joint Project*

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**Abstract**

Agents voluntarily contribute to a repeated joint project. We investigate the conditions for cooperation to be a renegotiation-proof and coalition-proof equilibrium before examining the influence of wealth (output share) inequality on the sustainability of cooperation. When shares are not equally distributed, cooperation requires agents to be more patient than under perfect equality. Beyond a certain degree of wealth inequality, full efficiency cannot be reached without redistribution. This model also explains the coexistence of one cooperating and one free-riding coalition. In this case, increased wealth inequality can have a positive or negative impact on the aggregate level of effort.

## 1 Introduction

Agents take part in a joint project to which they voluntarily contribute efforts. Each player gets a share of the output according to her wealth. On the one hand, we know that, if the game is played only once, the first-best optimum will be impossible to sustain[1], as deviation is a dominant strategy and the aggregate level of Nash equilibrium contributions is suboptimal. On the other hand, the Folk theorem teaches us that, provided people are not too impatient, new and more efficient equilibria can be reached thanks to the repetition of the game. In this paper, we want to investigate to what

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[1]It can if all the shares are concentrated into the hands of a single individual and efforts are perfectly substitutable.

extent inequality of wealth influences first-best sustainability in an infinitely repeated game. *Does introducing wealth inequality render cooperation more difficult to support?*

There exist numerous examples of voluntarily provided joint projects in the real world: voluntary provision to local public goods, collective action problems in management of environmental resources (forests, fisheries, irrigation schemes, pest and weed control), cooperatives, financial lobbying, defence alliances, etc. The literature in Industrial Organization is also related to theses issues, i.e. tacit collusion and to a lesser extent moral hazard in teams. Bardhan and Singh (2005) also evoke Velasco and the theoretical literature on the common pool problem in fiscal stabilization policy in Latin America.

Despite most of the literature on these issues uses a static game formalization, these situations are often better described as repeated games. One can indeed easily imagine that people interact repeatedly[2] and have to find ways to sustain cooperation between them, not knowing when this game is going to end. Our infinitely repeated game framework seems therefore quite realistic.

In one-shot games, Olson (1965), followed by several authors, argued that the effect of inequality was positive on collective action. He indeed claimed that if one single (or a few) agent has a great interest in the collective action, the good is more likely to be provided even if this user is the only one to bear its cost. Oslon brought up the two following insights: that contributions are positively related to wealth, which seems plausible, and that great inequality implies a great likelihood of success of collective action. A lot of papers show that the conditions for the latter result to remain valid are quite demanding. This is put forward, among others, in empirical studies by Bardhan (2000) and Dayton-Johnson (2000) but also in more theoretical papers such as Ray, Baland, Dagnelie (2007).
Nowadays, it seems that the effect of inequality - not necessarily of wealth - on efficiency or the aggregate level of effort is ambiguous, depending on the cost function (Banerjee et al., 2006). In a convincing empirical paper on collective action in Pakistanese communities, Khwaja (2006) finds an U-shaped relationship between inequality in the distribution of project returns or land ownership and maintenance, the aggregate level of efforts. It also appears that, in many cases, departing from an equalitarian distribution will hurt collective action or efficiency, by increasing the poor's incentives to free-ride or by allowing the rich and powerful to take over rents - as in the case of the Maharashtra sugar cooperatives (Banerjee et al., 2001). Then, deepening inequality can increase or decrease further the incentives of the beneficiaries from the redistribution to cooperate[3].

With the prospect of future interactions with the same people[4], arises the possibility

[2]It fits particularly well the case where people live in a small rural community and share resources.

[3]A comprehensive discussion on this issue can be found in Baland and Platteau (2003) from p. 161.

[4]This renders our framework different from reputation matching games à la Kandori (1992) where relationships are infrequent and "agents change their partners over time and dishonest behaviour against one partner causes sanctions by other members in the society".

of punishing undesired actions, which is conducive to cooperative outcomes. A few papers tackle the question of dynamic or repeated games. The former refers to a paper by Tarui (2007) investigating the influence of inequality in productivity, access to markets and credit into a dynamic intergenerational game of common property resource use. It takes into account how the resources of the commons endogenously evolve given the harvests by users in previous generations. According to the punishment used and the harvest sharing rule, Tarui shows that first-best sustainability may or not be affected by an increase of inequality.
As to Bardhan and Singh (2005), they explore the influence of wealth inequality on cooperation, sustained by trigger strategies, i.e. Nash reversion. In their model, agents are endowed with private capital which enters, with a complementary input, a constant return to scale Cobb-Douglas production function. To produce this complementary input, agents have to choose between a status quo technology which guarantees some level of output and a cooperative technology the fruits of which can be captured by one or more deviating players. They establish that, in this setting, inequality can affect cooperation and that redistribution can improve the welfare of the rich thanks to the greater possibility of cooperation.
This paper is also close to Itaya and Yamada (2003) who investigate the impact of income inequality on a repeated game of private provision of public goods with two players and renegotiation-proof equilibria. They also point out the negative effect of inequality on first-best sustainability.

Our research distinguishes itself from these two contributions to the literature by using a simpler model and by investigating how a renegotiation-proof and coalition-proof equilibrium will react after introducing a disequalizing change in the distribution of wealth.

This research is also very close to Vasconcelos (2005), a paper on tacit collusion in quantity setting supergames with asymmetric costs. It is quite interesting to transpose Vasconcelos' setting to our model. In both papers, inequality decreases the scope of cooperation sustainability, by increasing the discount factor of interest. Consistently with a large number of historical and empirical evidence, Vasconcelos' market shares are allocated according to the firms' production capacity, which affects marginal costs. He shows that the smallest firms are more prone to deviate from the collusive agreement, which fits our framework. As he uses optimal penal codes à la Abreu (1986, 1988) to sustain cooperation, while punishing, quite expectedly, the largest firms have the greatest incentives to deviate from the punishment. Here lies the main difference with our research[5]. As we resort to renegotiation-proof punishments, the cooperative agents do not suffer while punishing. They even profit by carrying out the punishment and are

[5] We also take into account deviations of credible coalitions whereas it is less relevant in Cournot competition.

therefore not tempted by deviations.

A common and easy solution considered in the literature to sustain cooperation is Nash reversion (Friedman, 1971) consisting in a permanent return to the Nash equilibrium after a single deviation. We will use it as a benchmark for our analysis. One of the main problems with Nash reversion is that, without being the harshest punishment, the punisher suffers from giving a punishment. As stated by Bernheim and Ray (1989) - collective dynamic inconsistency - and Farrell and Maskin (1989) - renegotiation-proofness - who introduced the concept of renegotiation-proofness in the literature[6], this renders the threat not credible. Everybody indeed anticipates that, ex post, the punishers and the punished will be tempted to renegotiate. Furthermore, after a single deviation, all the agents are stuck for ever in a pareto dominated equilibrium. It would be hard to believe that, in a repeated setting, agents fail to exploit the existing opportunities to reach the pareto frontier. Actually, this is rarely observed on the field as stated in Tarui (2007). The latter, indeed, evokes Ostrom (1990) who argues that many commons overcame occasional deviations. This indirectly supports the evidence that the punishments are only temporary, allowing a return to cooperation.

There lies the reason why we investigate this kind of punishments known to restrict the set of sustainable cooperative outcomes. From a prisoner's dilemma with two players to $n$ players, we add one layer of complexity as one has to take into account credible deviations[7] by coalitions of players.

In this work, we devise renegotiation-proof and coalition-proof punishments allowing to sustain cooperation as long as the agents are not too impatient. We consider the theoretical possibility that the first-best outcome is reached by checking that the discount factor compatible with the different conditions imposed by our punishment is smaller than one. We also show that the introduction of inequality under Nash reversion or punishments resisting to renegotiation and deviation by credible coalitions is detrimental to cooperation. In the presence of inequality of wealth, the agents have to be more patient not to deviate than under perfect equality of shares. We also demonstrate that, if $\gamma = 2$, the rich players - the ones able to cooperate - are always better off after giving part of their wealth to the poor - those whose share is so small that cooperation is too expensive for them - so that everyone can afford to cooperate.

In Section 2, we present our simple model. Nash reversion is addressed in Section 3. Then, in Section 4, we propose a renegotiation-proof and coalition-proof punishment scheme and investigate the influence of wealth inequality on the limit discount factor. Eventually, before concluding in Section 6 as to the negative influence of wealth in-

[6]Readers interested by this topic could also see the works of van Damme (1989) and Asheim.

[7]In some cases, a certain number of agents is required to make a deviation profitable. Hence, no one would like to deviate unless an agreement is reached. Once such an arrangement is found, it might still be interesting for one or more players to renege on the agreement and deviate further. This would render the deviation not credible, a case which we will not address in this paper.

equality in this setting, we characterize, in Section 5, the lowest share compatible with generalized cooperation and discuss the coexistence of one free-riding and one cooperating coalition and how redistribution can increase or decrease the total amount of effort put in the project. We also discuss how redistributing shares from the rich to the poor players can improve the welfare of everybody. The proofs are collected in an Appendix.

## 2 Repeated Joint Production with Shares

A group of $n$ agents decide to produce jointly and repeatedly a particular output. All the $i$ agents are identical except for, $\lambda_i$, their share in output which is also a measure of their wealth. $\boldsymbol{\lambda}$ is the vector of shares, $[\lambda_1, \lambda_2, \ldots, \lambda_n]$, the sum of which equals 1. Note that $\lambda_1 \leq \lambda_2 \leq \ldots \leq \lambda_n$. All the agents have the same discount factor, $\delta$, and therefore exhibit the same degree of impatience[8].

The output is the sum of the (nonnegative) efforts, $e_i$, of all the agents taking part in the project. One can note that under this modelling of the output, efforts are perfect substitutes[9] what, with respect to efficiency, favours an unequal distribution of shares. If $\lambda_i = 1$, the first-best optimum is indeed produced at each stage of the game what is not the case if efforts are less substitutable. Each agent putting effort in the project has to undergo an isoelastic convex cost, $e_i^{\gamma}$ with $\gamma > 1$ [10].

Individual payoffs, $\pi_i$, have therefore the following form:

$$\pi_i = \lambda_i \sum_i e_i - e_i^{\gamma} \tag{1}$$

By maximizing the social surplus, we get that the cooperative level of effort, $e_i^C$, must be equal to $\gamma^{\frac{-1}{\gamma-1}}$, which is independent from the distribution of wealth.
If this game is played once, then every agent is going to maximize her own payoff, given the efforts contributed by the other agents.

$$\max_{e_i} \lambda_i (e_i + \sum_{j \neq i} e_j) - e_i^{\gamma} \tag{2}$$

In this case where everyone deviates from the cooperative level of effort and produces

[8]If one introduces inequality in the distribution of discount factors assuming that the poor are less patient than the rich, for instance by endogenizing $\delta$ w.r.t. $\lambda$, cooperation is even harder to sustain and our conclusions easier to reach.

[9]For a discussion on the influence of inequality on joint projects when efforts are not perfect substitutes in a one-shot game, see Ray, Baland and Dagnelie (2007).

[10]If $\gamma = 1$, surplus maximization is unbounded.

$e_i^N \equiv \gamma^{\frac{-1}{\gamma-1}} \lambda_i^{\frac{1}{\gamma-1}}$, the outcome is known to be pareto dominated by the first-best optimum[11].

The game described above is similar to a prisoner's dilemma with $n$ agents and continuous strategies. Under generalized cooperation, everyone has to produce the same amount of effort which proves to be costlier for the less endowed agents. Inequality creates tensions in the sense that the net profit from cooperation is lower for the poor and that deviation is more attractive to them.

LEMMA **1** *The agent the most tempted by deviation is always the one with the lowest share.*

When we introduce wealth inequality, the cooperation payoff decreases faster than the deviation payoff which makes deviation more profitable for the poor. This tends to confirm Olson's hypothesis as to the exploitation by the poor.

If $\pi_i^{C^*} < \pi_i^C$ - where $\pi^{C^*}$ is the deviation profit[12] which depends on the number of deviating players and $\pi^C$ is the cooperation profit[13] - deviating from the cooperation effort is not profitable. There is therefore no need of punishment in those cases. In this paper, we are not paying attention to those not credible deviations. For deviating to be interesting, the following condition must be fulfilled:

$$\frac{\lambda_i \gamma (n_D - \sum_{n_D} \lambda_i^{\frac{1}{\gamma-1}})}{1 - \lambda_i^{\frac{\gamma}{\gamma-1}}} < 1$$

with $n_D$ being the number of deviating players. Under a perfectly equalitarian distribution of shares (i.e. $\lambda_i = \frac{1}{n}$), it simplifies to :

$$\gamma \frac{n_D}{n} < \frac{1 - n^{\frac{-\gamma}{\gamma-1}}}{1 - n^{\frac{-1}{\gamma-1}}} \equiv n_D < \frac{n - n^{\frac{-1}{\gamma-1}}}{\gamma(1 - n^{\frac{-1}{\gamma-1}})}$$

which, if $\gamma$ tends towards 1, is true even when $n_D = n - 1$. For the inequality to be satisfied, the number of deviating players must decrease when $n$ and $\gamma$ rise. When $\gamma = 2$, it is true as long as $n_D < \frac{n+1}{2}$. If $\gamma > 2$, when $n$ is even, from 6 players, the condition is not fulfilled for all $\gamma$ with $n_D = \frac{n}{2}$. With $n$ odd, the problems begin when there are 9 agents and $n_D = \frac{n+1}{2}$. It is to be noted that the condition is always satisfied if $n_D = 1$. While cooperating produces a big surplus, if too few cooperate and too many deviate, the small surplus is divided among too many deviating players for deviating to remain profitable.

[11] It means that $e_i^N = e_i^C \lambda_i^{\frac{1}{\gamma-1}}$ with $e_i^N < e_i^C$ as, if $\lambda_i < 1$, $\lambda_i^{\frac{1}{\gamma-1}} < 1$ for all $\gamma$. In the following analysis, $\gamma^{\frac{-1}{\gamma-1}}$ is often put in evidence, which implies that $e_i^C = 1$ and $e_i^N = \lambda_i^{\frac{1}{\gamma-1}}$.

[12] $\pi_i^{C^*} = \gamma^{\frac{-1}{\gamma-1}} \lambda_i (n - n_D + \sum_{n_D} \lambda_i^{\frac{1}{\gamma-1}} - \lambda_i^{\frac{1}{\gamma-1}} \gamma^{-1})$, where $n - n_D = n_C$ cooperate but $n_D$ deviate.

[13] $\pi_i^C = \gamma^{\frac{-1}{\gamma-1}} (\lambda_i n - \gamma^{-1})$

## 3 Nash Reversion

In this section, we will temporarily not address the problems posed by renegotiation and consider the following punishment: Once a coalition has deviated, everybody produces the Nash level of effort for ever. For cooperation to be sustainable, the discount factor must respect the following condition:

$$(1-\delta_N)\pi^{C^*} + \delta_N \pi^N < \pi^C \Rightarrow \delta_N > \frac{\pi^{C^*} - \pi^C}{\pi^{C^*} - \pi^N}$$

where $\pi^N$ is the Nash profit[14], corresponding to generalized deviation.

If $\frac{\pi^{C*} - \pi^C}{\pi^{C*} - \pi^N} < 1$, it is theoretically possible to support cooperation. As long as players have a discount factor greater than $\delta_N$ and smaller than 1, Nash reversion can be used as a threat against deviation. Hence, the necessary condition for cooperation to become sustainable is :

$$\pi_i^C > \pi_i^N \equiv \frac{\lambda_i \gamma (n - \sum_i \lambda_i^{\frac{1}{\gamma-1}})}{1 - \lambda_i^{\frac{\gamma}{\gamma-1}}} > 1 \tag{3}$$

The bigger the difference $\pi_i^C - \pi_i^N$, the smaller $\delta_N$ and therefore the easier cooperation can be sustained.

One can see that, if the distribution of wealth is perfectly equalitarian, as long as the agents have a discount factor smaller than 1 and greater than $\delta_N$, cooperation can always be sustained by Nash reversion. The condition in equation (3) is indeed always verified and becomes :

$$\gamma > \frac{1 - n^{\frac{-\gamma}{\gamma-1}}}{1 - n^{\frac{-1}{\gamma-1}}} \tag{4}$$

PROPOSITION **1** *Introducing inequality among agents renders the condition to sustain cooperation with Nash reversion more difficult to fulfill.*

As cooperation can always be sustained with Nash reversion when the distribution is equalitarian and deviation is a dominant strategy for the agents for which inequality renders $\pi_i^C < \pi_i^N$, one can expect a negative effect of inequality on cooperation. Following the introduction of inequality, the agents losing from the redistribution of wealth, i.e. $\lambda_i < n^{-1}$, have to be more patient than before not to enter a deviation phase. All this means that increasing wealth inequality makes cooperation harder and harder to sustain - as the limit discount factor, $\delta_N$, rises - up to a point where inequality is such that, whatever the impatience degree of the agent with a low share, cooperation is not possible any more.

[14] $\pi_i^N = \gamma^{\frac{-1}{\gamma-1}} \lambda_i (\sum_i \lambda_i^{\frac{1}{\gamma-1}} - \lambda_i^{\frac{1}{\gamma-1}} \gamma^{-1})$

# 4 A Renegotiation-Proof and Coalition-Proof Equilibrium

As mentioned before, Nash reversion suffers from several flaws. That is why we want to turn to a renegotiation-proof and coalition-proof equilibrium preventing credible coalitions to deviate from the first-best production level. We also want this equilibrium to allow a return to cooperation after the punishment phase.

The case with $n$ players is a bit different to two players as there can be deviations of coalitions. Nonetheless, the latter fits in with the framework set out below. As suggested by van Damme (1989) in a prisoner's dilemma with discrete strategies, cooperation can be sustained if as soon as player 1 has deviated, player 2 deviates until player 1 cooperates.

The same idea is used to devise our punishment scheme. After a deviation of a coalition of one or more players, the cooperative players enter a punishment phase during which they produce a retaliation quantity of effort. They keep playing this level of effort as long as the cheaters - the ones who deviated in the normal phase - have not played the punishment level of effort. During this punishment, everybody has the incentives to play accordingly to the scheme - what ensures subgame perfectness - and the punishers get at least as much as when everybody cooperates and produces the first-best optimum - what guarantees renegotiation-proofness.

We can immediately restrict the length of the punishment phase thanks to the following lemma.

LEMMA **2** *Aiming at the smallest discount factor compatible with a renegotiation-proof and coalition-proof punishment limits the length of the punishment phase to one period.*

A multi period punishment would have two effects, ex ante, it would make deviation less attractive but as the punishment is harsher, ex post, it would increase the incentives to deviate from conforming to the punishment. As will become clearer below, there is a trade-off between these two effects. In this particular case, the latter effect would be dominating as the punishment level of effort is high enough for the punishers to be willing to punish, given the requirement of renegotiation-proofness.

The discount factor allowing cooperation to be sustained must respect the following condition:

$$(1-\delta)\pi^{C^*} + \delta(1-\delta)\pi^P + \delta^2\pi^C < \pi^C$$
$$\Rightarrow \delta_P > \frac{\pi^{C^*} - \pi^C}{\pi^C - \pi^P} \tag{5}$$

$\delta_P$ must be smaller than 1 which implies that $2\pi^C - \pi^{C^*} - \pi^P > 0$, with $\pi^P$ being the payoff obtained by an agent during her punishment. Meeting this condition will prevent a subcoalition of players to alternate between deviating and being punished every other period. If the punishment effort is fixed at the level of the cooperative effort, this condition is always verified as $2\pi^C - \pi^{C^*} - \pi^P > 0$ boils down to equation (4). Hence we know that, if the punishment effort to put in is greater than during cooperation, $\delta_P$ decreases.

For a punishment to be renegotiation-proof and coalition-proof, four other conditions need to be satisfied.

**Condition 1** *The payoff of the punished must be greater when conforming to their punishment than when deviating.*

$$(1-\delta)\pi^P + \delta\pi^C > (1-\delta)\pi^{P^*} + \delta(1-\delta)\pi^{P'} + \delta^2\pi^C$$
$$\Rightarrow \delta_{cond1} > \frac{\pi^{P^*} - \pi^P}{\pi^C - \pi^{P'}} \tag{6}$$

with $\pi^{P^*}$ being the profit obtained when deviating from undergoing the punishment and $\pi^{P'}$, the profit received, while incurring the penalty, by the subcoalition of agents who deviated from the punishment. When $n_{D^*} = 1$, $\delta_{cond1}$ reaches its minimum which therefore makes Condition 1 the most easily fulfilled. The benefit from deviating from the punishment is by far outweighed by the burden of the penalty for which the single deviator has to compensate all the cooperating players. If the size of the deviating subcoalition rises, $\pi^{P^*}$ decreases whereas $\pi^{P'}$ rises much faster which means that $\delta_{cond1}$ also rises. It is therefore expected that $\arg\max_{n_{D^*}\in(1,n_D)} \delta_{cond1} = n_D$.

The following two conditions ensure that all the punishers are willing to conform to the punishment phase.

**Condition 2** *The payoff of the punishers must be greater when conforming than when deviating from punishing and then conforming.*

If $\pi^{1/P}$ and $\pi^{1/P^*}$ represent respectively the profit from punishing and from deviating from giving the punishment, we get:

$$(1-\delta)\pi^{1/P} + \delta\pi^C > (1-\delta)\pi^{1/P^*} + \delta\pi^C \Rightarrow \pi^{1/P} > \pi^{1/P^*}$$

In case of perfect equality of wealth among agents, we have:

$$\gamma\frac{n_{C^*}}{n}(C - C^*) > (C^\gamma - {C^*}^\gamma) \tag{7}$$

The idea behind this condition is that it could be more interesting, for the punishing players, to skip the punishment phase and go back directly to cooperation. We are going to check which values of $C$ and $C^*$ are compatible with a renegotiation and coalition-proof punishment. $C^*$ is the level of effort put in when deviating from giving a punishment. Equation (7) must hold for $n_{C^*} = 1$, while if it does not hold when $n_{C^*} > 1$

we have to ensure that all these deviating coalitions are not credible. This is done in Condition 3.

One could imagine that the simplest form of punishment would be that the punishers do not produce for one period while the deviators are constrained to put in such a level of effort that the punishers get at least the cooperative payoff. However, as long as $C < e^N$, for very high values of $\gamma$, it could be interesting for one punisher to deviate and produce the cooperative level of effort[15]. Once $C$ is fixed at the Nash level of effort, it is never interesting for one punisher[16] to deviate from the punishment scheme. Once that $C$ is fixed to $e_i^N$, it is easy to prove that:

$$\frac{\partial \delta_{cond1}}{\partial n_{D^*}} > 0 \tag{8}$$

Hence, we have to focus on the case where Condition 1 is the hardest to satisfy, i.e. $n_{D^*} = n_D$, which gives $\delta_{cond1} > \frac{\pi^N - \pi^P}{\pi^C - \pi^P}$. As it is easy to see that $\frac{\partial \delta_{cond1}}{\partial \pi^P} < 0$ [17], we know that the minimum of $\delta_{cond1}$ is reached when the punishment is also fixed at its minimum.

All that has been said so far allows us to remark that $\pi^N$ is again a focal point. If the cooperative agents return to putting in the Nash effort, the threat point of the game is infinite repetition of the Nash equilibrium. The deviators can indeed renege for ever on the punishment and also produce the Nash effort. Hence we know that $\pi_i^C > \pi_i^N$ is again to satisfy.

For this setting to be coalition-proof, we must now ensure that no deviating coalition of punishers is credible. For the deviation to be credible, no one should have one's interest in further deviating from the deviating coalition. We hence turn to the next condition.

**Condition 3** *For the equilibrium to be coalition-proof, no deviation of punishers should be credible, i.e.* $\pi^{1/P^*} < \pi^{1/P^{**}}$.

If $\lambda_i = \frac{1}{n}$ and $n_{C^{**}} = 1$ [18], it is equivalent to:

$$\frac{\gamma}{n} < \frac{(1 - n^{\frac{-\gamma}{\gamma-1}})}{(1 - n^{\frac{-1}{\gamma-1}})} \tag{9}$$

[15] As showed by $\lim_{\gamma \to +\infty} \frac{\gamma}{n}(\beta n^{\frac{-1}{\gamma-1}} - 1) - (\beta^\gamma n^{\frac{-\gamma}{\gamma-1}} - 1) < 0$ when $\beta < 1$. For the particular case of $C = 0$, it is easy to check that the condition $C^* > (\frac{\gamma}{n})^{\frac{1}{\gamma-1}}$ is not satisfied for many values of $\gamma, n$.

[16] A coalition of $n_{C^*} > 1$ players could be tempted to deviate from giving the punishment. This prevents this scheme to be strong Nash.

[17] $\frac{\partial \delta_{cond1}}{\partial \pi^P} = \frac{\pi^N - \pi^C}{(\pi^C - \pi^P)^2}$ which is negative as long as $\pi^C > \pi^N$.

[18] If one player wishes to deviate from the deviating coalition, the latter is not credible.

For all $\gamma$ and $n$, this condition is always fulfilled which means it is always more interesting for one player to further deviate from the deviating coalition by producing the Nash level of effort[19]. Condition 3 is always fulfilled and no deviation of punishers is therefore credible.

It is to be noted that, if all the punished conform to the punishment phase and produce a high level of effort, no coalition of punishers is willing to deviate. A fortiori, if a subcoalition of punished deviates from the punishment phase - and therefore produces less -, no subcoalition of punishers is tempted to skip this phase. Given the punished have opposite duties than the punishers, no deviation of a mixed coalition is credible.

The following condition ensures the punishers are not going to renegotiate the punishment scheme.

**Condition 4** *The payoff from punishing a deviating coalition must be greater or equal than the payoff from generalized cooperation, i.e.* $\pi^{1/P} \geq \pi^{C}$.

It means that, if $\lambda_i = \frac{1}{n}$,

$$\underline{P} \geq [n - n_C C - n\gamma^{-1}(1 - n^{\frac{-\gamma}{\gamma-1}})]\frac{1}{n_D} \tag{10}$$

Now that the conditions to satisfy are stated, we want to find the punishment producing the lowest discount factor compatible with generalized cooperation, $\underline{\delta}$. We have therefore to fix the effort level corresponding to the punishment, $P$, so that the couple $(\delta_P, \delta_{cond1})$ is the lowest possible and, in any case, smaller than one. As our punishment scheme has to simultaneously respect the conditions expressed in equations (5) and (6), we have to find the $\max(\delta_P, \delta_{cond1})$. To obtain the punishment corresponding to the highest degree of impatience, we have to determine $P$ such that we get $\underline{\delta} \equiv \min\max(\delta_P, \delta_{cond1})$.
Comparing $\delta_P$ and $\delta_{cond1}$ boils down to comparing $\pi^{C^*} - \pi^C$ and $\pi^N - \pi^P$, in the limit case where $n_{D^*} = n_D$, as the denominator of these fractions is the same. After simplification,

$$\pi^{C^*} - \pi^C \leq \pi^N - \pi^P \tag{11}$$

becomes:

$$-\gamma^{-1}P^{\gamma} + n_D n^{-1} P - n_D n^{-1} + \gamma^{-1} \leq 0 \tag{12}$$ [20]

The root of interest in equation (12) is $P = 1$, with $P = 1$ meaning that the punished ones have to produce the cooperative level of effort while undergoing their punishment. Hence we know that when $P = 1$, $\underline{\delta} = \delta_P = \delta_{cond1}$. We also know that as soon as $P > 1$, we have to minimize $\pi^N - \pi^P$ and hence $P$ to get $\underline{\delta}$. It means that the minimal

[19] It is not surprising as $e_i^N$ is the best response of player $i$, being the solution to equation (2).

[20] It can also be expressed as follows: $-n_D n(1-P) + \gamma^{-1}(1-P^{\gamma}) \leq 0$.

punishment compatible with renegotiation-proofness and coalition-proofness is $\underline{P}$ as long as it is greater than 1. As long as the parameters of equation (10) produce a $\underline{P} < 1$, resorting to such a punishment effort will not prevent deviations. The penalty incurred by the deviators would be too small to deter them from alternating between deviating and being punished. In this case, the punishment must be to put in the cooperative level of effort, i.e. $P = 1$ [21]. On the other hand, if $P < \underline{P}$, the punishment is not renegotiation-proof.
We have therefore to find $P \equiv \max(1, \underline{P})$ and turn to the following equation.

$$P \geq 1 \equiv \frac{n_C}{n}\gamma \geq \frac{1 - n^{\frac{-\gamma}{\gamma-1}}}{1 - n^{\frac{-1}{\gamma-1}}} \tag{13}$$

Taking into account the condition for deviation to be profitable, i.e. $\pi^{C^*} - \pi^C > 0$, the only case where equation (13) is not verified, with $\gamma \geq 2$, is when $n_C = n_D = \frac{n}{2}$. In the latter case, $\underline{P}$ is too small a punishment and the punished have to put in the cooperative level of effort, exactly as in the two players game put forward by van Damme (1989). Note that this particular case can happen only if $n$ is even. As to the case of $\gamma < 2$, most of the credible deviations must be deterred by $\underline{P} = 1$, since a low $\gamma$ makes $\underline{P}$ too mild a punishment.

Considering we took into account the different conditions imposed by our punishment scheme when $\lambda_i = n^{-1}$, we are equipped with the parameters of our punishment, $P$ and $C$. As $\delta_{cond1}$ must be smaller than 1 for cooperation to be a renegotiation-proof and coalition-proof equilibrium, $\pi_i^N$ must be smaller than $\pi_i^C$. It is the same binding constraint as with Nash reversion and we know from equation (4) that it is always true. This allows us to put forward the following proposition:

PROPOSITION **2** *As long as $\delta_{cond1} < \delta < 1$, $\lambda_i = n^{-1}$ and the game is infinitely repeated*[22]*, it is possible to sustain cooperation with a renegotiation-proof and coalition-proof punishment.*

So far, we have showed that, under a perfectly equalitarian distribution of wealth, it is possible to use a renegotiation-proof and coalition-proof punishment scheme to sustain generalized cooperation. One can remark that, with $\gamma$ known and a perfectly observable and certain output, this equilibrium requires particularly little information. It can be completely decentralized as, after deviation, the punished know exactly which level of effort to provide.

We now want to investigate how introducing wealth inequality influences the way the first-best optimum can be supported. The punishment is, as expected, very similar

[21] As the factor $\gamma^{\frac{-1}{\gamma-1}}$ is always put in evidence in this analysis.
[22] It remains true if the agents do not know when the game ends.

to the case of perfect wealth equality. If we introduce inequality, $\pi^{C^*} - \pi^C \leq \pi^N - \pi^P$ becomes:

$$\lambda_j(-n_D + \sum_{n_D} P_j) + \gamma^{-1}(1 - P_j^{\gamma}) \leq 0 \tag{14}$$

in which the only case where the $\sum_{n_D} P_j$ is insufficient to satisfy equation (14), when $\gamma \geq 2$, occurs again when $n_D = \frac{n}{2}$. In this case, $\sum_{n_D} P_j$ must be equal to $n_D$ and hence $P_j = 1$. As in the equalitarian case, the latter punishment effort is much more frequent when $\gamma < 2$. For all the other cases, equation (10) becomes:

$$\underline{P}_j \geq \frac{\lambda_j}{\sum_{n_D} \lambda_j}[n - \sum_{n_C} \lambda_i^{\frac{1}{\gamma-1}} - \gamma^{-1} \frac{\sum_{n_C} \lambda_i^{-1}(1 - \lambda_i^{\frac{\gamma}{\gamma-1}})}{n_C}]$$

While, as each punisher must receive $\pi_i^{1/P} = \pi_i^C$, all the deviators have to put in $\underline{P}_j$ so that:

$$\pi_i^{1/P} = \gamma^{\frac{1}{1-\gamma}} \left[\lambda_i(\sum_{n_C} \lambda_i^{\frac{1}{\gamma-1}} + \sum_{n_D} \underline{P}_j) - \gamma^{-1} \lambda_i^{\frac{\gamma}{\gamma-1}}\right]$$

where $\forall i \in n_C : \sum_{n_D} \underline{P}_j = n - \sum_{n_C} \lambda_i^{\frac{1}{\gamma-1}} - \gamma^{-1} \lambda_i^{-1}(1 - \lambda_i^{\frac{\gamma}{\gamma-1}})$.

We showed that, under a perfectly equalitarian distribution of shares, our punishment scheme prevents all the agents from deviating from cooperation. We also know that inequality is detrimental to cooperation as it is possible that deviation becomes a dominant strategy for poorly endowed agents - i.e. if $\pi_i^C < \pi_i^N$. However, even in less extreme cases, we can state the following proposition.

PROPOSITION **3** *After introducing inequality, the agents losing from the disequalizing change in the distribution of shares have to be more patient than before to produce the efficient level of effort when the punishment is renegotiation-proof and coalition-proof.*

As in the case of Nash reversion, introducing inequality of wealth renders first-best efficiency more difficult to support with a renegotiation-proof and coalition-proof punishment.

Now that we have characterized the minimal discount factor with those two kinds of punishment, we can compare them. It is easy to show that $\delta_N \leq \delta_{cond1}$ as, after rearranging and symplifying, we get equation (11).

# 5 Redistribution and Cooperation

In this section, we first try to characterize the lower bound of theoretical cooperation, i.e. when $\delta$ tends towards 1. Note again that the further $\left(\pi_i^C - \pi_i^N\right)$ is from 0, the lower $\delta$, which increases the scope for cooperation. Then, we investigate the issue of redistribution.

### 5.1 Characterization of the lowest share compatible with generalized cooperation

As the utmost condition to satisfy for cooperation to be sustainable is $\pi_i^C > \pi_i^N$, it is possible to characterize the lowest share compatible with generalized cooperation, $\lambda_{min}$. To see this, let us define several distributions of shares:

$$\begin{array}{l} \underline{\boldsymbol{\lambda}} \equiv \underline{\lambda}_1 = \ldots = \lambda_{n-1} < \lambda_n \\ \tilde{\boldsymbol{\lambda}} \equiv \tilde{\lambda}_1 = \ldots = \lambda_{n-2} < \lambda_{n-1} < \lambda_n \\ \hat{\boldsymbol{\lambda}} \equiv \hat{\lambda}_1 = \ldots = \lambda_{n-2} < \lambda_{n-1} = \lambda_n \\ \bar{\boldsymbol{\lambda}} \equiv \bar{\lambda}_1 < \lambda_2 = \ldots = \lambda_n \end{array}$$

The lowest share compatible with cooperation depends on the convexity parameter of the cost term, $\gamma$. Therefore, we get:

$$\lambda_{min} \geq \begin{cases} \bar{\lambda}_1 & \text{if } \gamma < 2 \\ -(n-1) + \sqrt{(n-1)^2 + 1} & \text{if } \gamma = 2 \\ \underline{\lambda}_1 & \text{if } \gamma > 2 \end{cases} \tag{15}$$

One can remark that, if $\gamma = 2$, $\lambda_{min}$ does not depend on the distribution of the other shares. In case some players can not afford to cooperate, a subcoalition of players can cooperate and $\lambda_{min} \geq -[n - (n_D + 1) + \sum_D \lambda_D] + \sqrt{[n - (n_D + 1) + \sum_D \lambda_D]^2 + 1}$.

### 5.2 Redistribution

All the players know that, whatever the punishment strategy we use, if $\pi_i^C < \pi_i^N$, the best strategy for player $i$ is to deviate at each period of the game. The first-best optimum can not be attained any more but cooperation can still be sustained for a subset of players whose cooperation profit is greater than the Nash profit[23].

**Observation 1** *The only influence of the agents not cooperating because of a low* $\underline{\lambda}_j$ [24] *is to diminish the share of the cooperators.*

Taking into account the poorly endowed agents, equation (3) becomes

$$\frac{\lambda_i \gamma (n_C - \sum_{i \neq j}^{n_C} \lambda_i^{\frac{1}{\gamma-1}})}{1 - \lambda_i^{\frac{\gamma}{\gamma-1}}} > 1$$

[23] This can be the case if we suppose that the rich players, knowing that the deviators are so poor that they cannot afford to cooperate, do not turn to punishing the deviating players.

[24] $\sum_j \underline{\lambda}_j = \underline{\lambda}_{inflim} \Rightarrow \sum_{i \neq j}^{n_C} \lambda_i = 1 - \underline{\lambda}_{inflim}$

This inequality is never verified if $n_C = 1$. It means that, if more than one agent gets a positive share, producing the first-best level of effort requires at least two agents to get a big enough share[25], i.e. a share such that $\pi_i^C > \pi_i^N$. The number of people whose shares add up to a low $\underline{\lambda}_{inflim}$ does not influence cooperation. At the same time, there can coexist one cooperating and one free-riding coalition.

As we know that cooperators[26] put in an effort of 1 and deviators contribute $\lambda^{\frac{1}{\gamma-1}}$, the total level of effort put in the project is $\sum_i e_i = n_C + \sum_{n_D} \lambda^{\frac{1}{\gamma-1}}$. It is therefore possible to compare the different distributions with respect to the total level of effort contributed to the project.

If we redistribute[27] from a deviator to a cooperator, it unambiguously decreases the aggregate level of effort. In this case, increasing wealth inequality has a negative impact on the total level of effort. While if the redistribution lowers the wealth of a poor player to the benefit of another poor to such an extent that he becomes rich enough to cooperate, it increases the total sum of contributions. Increasing inequality can therefore have a positive effect on the amount contributed. We then get a U-shaped relationship between inequality and the aggregate level of effort. Eventually, if we redistribute among poor agents who keep on producing the Nash level of effort, the effect depends on $\gamma$, the isoelastic cost parameter. When $\gamma < 2$, $()^{\frac{1}{\gamma-1}}$ is convex which means that every disequalizing redistribution among the poor increases the aggregate level of effort. If $\gamma > 2$, the function is concave and the reverse is true. While the $\gamma = 2$ case is neutral as to the effect of redistribution on the amount of efforts provided in the project.

This being said, we now want to investigate whether it would be profitable for the rich agents to redistribute part of their wealth to the less endowed so that the latter can afford to cooperate. Once the poor get a share such that their cooperation profit is at least equal to their deviation profit, they are expected to produce the efficient level of effort. The benefits from cooperation are such that, in some cases, they outweigh the loss of welfare following the redistribution of wealth of the rich agents. It allows us to put forward the following observation.

**Observation 2** *If $\delta$ is close enough to 1, when $\gamma = 2$, it is always interesting for the rich players to redistribute part of their wealth to the ones who can not afford to cooperate.*

Proposition 2 remains true at the neighbourhood of 2 and, depending on the initial distribution of shares, may hold for higher values of $\gamma$. But with $\gamma$ rising, it is harder and harder to fulfill the conditions for redistribution to be profitable.

[25]It implies that, if all the agents but one have a share such that $\pi_i^C < \pi_i^N$, the richest player (whose $\lambda_n < 1$) will produce the deviating level of effort.

[26]If $\gamma^{\frac{-1}{\gamma-1}}$ is put in evidence. More rigorously, $\sum_i e_i = \gamma^{\frac{-1}{\gamma-1}} \left( n_C + \sum_{n_D} \lambda^{\frac{1}{\gamma-1}} \right)$.

[27]It has exactly the same effect as comparing different distributions of shares.

## 6 Conclusion

We showed that, in this particular model, cooperation can be supported under Nash reversion or a renegotiation-proof and coalition-proof punishment. We also demonstrated that introducing inequality of wealth among players increases the discount factor compatible with sustainable cooperation, reducing the scope for cooperation. Once inequality has been introduced, the agents involved in the repeated project have to be more patient than before. Hence, we demonstrated that inequality is, in this game, detrimental to generalized cooperation, the efficient outcome.

Inequality can be such that some agents can not afford to produce the efficient level of effort. Our model can therefore also explain the coexistence of well endowed players providing a high level of effort and poor agents who can only put in the Nash level of effort. This, in a way, complies with Olson's hypothesis that contributions are positively related to wealth.

A comparison of several wealth distributions with a coexistence of cooperators and deviators shows that increasing wealth inequality can have a positive or negative impact on the aggregate level of effort depending on the number of cooperators and the cost parameter. Hence, wealth inequality can have a U-shaped relationship with the aggregate level of effort, as regularly seen in case studies.

We eventually proved that, in some cases, - particularly around $\gamma = 2$ - it can be profitable for the rich agents to redistribute part of their wealth to the poor players so that they can afford to cooperate at each period of the game.

# Appendix

### Proofs

**Proof of Lemma 1:**

$$\pi^{C^*} - \pi^C = \gamma^{\frac{1}{1-\gamma}} \left[\lambda_i(\sum_{n_D} \lambda^{\frac{1}{\gamma-1}} - n_D) - \gamma^{-1}(\gamma^{\frac{\gamma}{\gamma-1}} - 1)\right]$$

As $\frac{\partial(\pi^{C^*} - \pi^C)}{\partial \lambda_i} < 0$, the premium from deviating rises when the share declines. ■

**Proof of Equation (4)**
After rearranging equation 4, we get:

$$(\gamma - 1) - n^{\frac{-1}{\gamma-1}}(\gamma - n^{-1}) > 0$$

Taking alternatively the limit of this expression with $\gamma$ towards 1 and $+\infty$, we get that the first term is always positive and greater than the second one. It makes our result. ■

**Proof of Proposition 1**

We are going to compare two discount factors compatible with cooperation, first under a perfectly equalitarian distribution of shares, then after introducing a disequalizing change in the distribution. As discount factors and ease to sustain cooperation vary in opposite directions, we are done if we can prove that introducing inequality in the distribution of shares makes the discount factor rise.

As long as $\delta_N < 1$, it is theoretically possible to sustain cooperation. We know that the bigger $\pi_i^C - \pi_i^N$, the lower $\delta_N$.

$$\pi_i^C - \pi_i^N = \gamma^{\frac{1}{1-\gamma}}\left[\lambda_i(n - \sum_i^n \lambda^{\frac{1}{\gamma-1}}) - \gamma^{-1}(1 - \gamma^{\frac{\gamma}{\gamma-1}})\right]$$

As $\frac{\partial(\pi^C - \pi^N)}{\partial\lambda_i} > 0$, a decrease in $\lambda_i$ lowers the gain from cooperation and therefore increases the incentives to deviate and the limit discount factor, $\delta_N$. ■

**Proof of Lemma 2**

Let us assume the length of the punishment is $t$ periods with $t \in [1, \ldots, T]$. If we compare the different $\delta$ corresponding to equations (5) and (6), we get:

- ex ante, no one wishes to deviate

$$\sum_{t=1}^{T} \delta_P^t > \frac{\pi^{C^*} - \pi^C}{\pi^C - \pi^P} \Rightarrow \delta_{P1} > \ldots > \delta_{PT}$$

- The payoff of the punished must be greater when conforming to their punishment than when deviating.

$$\delta_{cond1-t} > (\frac{\pi^{P^*} - \pi^P}{\pi^C - \pi^P})^{\frac{1}{t}} \Rightarrow \delta_{cond1-1} < \ldots < \delta_{cond1-T}$$

To get the equilibrium compatible with the biggest impatience of the agents, we have to find the lowest $\delta$, $\underline{\delta} \equiv \min\max(\delta_{Pt}, \delta_{cond1-t})$ with $t$ being the number of periods of the punishment.

If $\delta_{P1} \leq \delta_{cond1-t}$, we are done as this couple would be smaller than any other one in the following general ordering: $\delta_{Pt} < \ldots < \delta_{P1} \leq \delta_{cond1-1} < \ldots < \delta_{cond1-t}$.

It therefore boils down to prove that $\pi^{C^*} - \pi^C \leq \pi^{P^*} - \pi^P$ which is the case when $P \geq 1$ as explained after equation (12). ■

**Proof of Equation (8)**

Taking into account that $\delta_{cond1} < 1$ and that $\pi^C - \pi^{P'} > 0$, and rearranging, we get

$\pi^{P^*} + \pi^{P'} - \pi^C - \pi^P < 0$. Our strategy is to focus on the terms varying with $n_{D^*}$ to get the simplest expression of the derivative.
Considering the punishment level of effort of equation (10), we get respectively for $P$ and $P'$:

$$P = \left[n - n_C n^{\frac{-1}{\gamma-1}} - \gamma^{-1} n(1 - n^{\frac{-\gamma}{\gamma-1}})\right] \frac{1}{n_D}$$
$$P' = \left[n - (n - n_{D^*}) n^{\frac{-1}{\gamma-1}} - \gamma^{-1} n(1 - n^{\frac{-\gamma}{\gamma-1}})\right] \frac{1}{n_{D^*}}$$

We rewrite $\delta_{cond1} < 1$ as follows

$$\begin{aligned}
&2n^{-1} - 2\gamma^{-1}(1 - n^{\frac{-\gamma}{\gamma-1}}) - \gamma^{-1} n^{\frac{-\gamma}{\gamma-1}} \\
&-n_{D^*}\left[\frac{n^{-1}}{n_D}\left(n - n_C n^{\frac{-1}{\gamma-1}} - \gamma^{-1} n(1 - n^{\frac{-\gamma}{\gamma-1}})\right) - n^{-1} n^{\frac{-1}{\gamma-1}}\right] \\
&-\gamma^{-1}\left[\frac{1}{n_{D^*}}\left[n - (n - n_{D^*}) n^{\frac{-1}{\gamma-1}} - \gamma^{-1} n(1 - n^{\frac{-\gamma}{\gamma-1}})\right]\right]^\gamma - \pi^C - \pi^P < 0
\end{aligned}$$

The derivative of the latter expression with respect to $n_{D^*}$ is:

$$-n^{-1}(P - n^{\frac{-1}{\gamma-1}}) + (P')^{\gamma-1} \frac{1}{n_{D^*}} (P' - n^{\frac{-1}{\gamma-1}}) > 0$$

As the second term is unequivocally greater than the first one, we prove our result. ■

**Proof of Equation (9)**
The scheme of this proof follows the proof of equation 4, what gives:

$$(\gamma - n) - n^{\frac{-1}{\gamma-1}}(\gamma - n^{-1}) > 0$$

Taking alternatively the limit of $\gamma$ towards 1 and $+\infty$, we get that the first term is always positive and greater than the second one. It makes our result. ■

**Proof of Proposition 3**
As, in the limit case where $n_{D^*} = n_D$, the condition for cooperation to be sustainable becomes $\pi_i^C > \pi_i^N$, the proof of Proposition 3 boils down to the proof of Proposition 1. ■

**Proof of Equation (15):**
As the condition for one agent to cooperate is $\pi_i^C - \pi_i^N > 0$, the bigger the positive difference, the easier the cooperation. We can therefore compare equation (3) [28], with

[28] $\frac{\lambda_i \gamma (n - \sum_i \lambda_i^{\frac{1}{\gamma-1}})}{1 - \lambda_i^{\frac{\gamma}{\gamma-1}}} > 1$

two different distributions of $\boldsymbol{\lambda}$ to see which one will be more favourable to cooperation. If $\lambda_1$ is compatible with cooperation such that equation (3) is verified then all the 'less compatible with cooperation' distributions of shares will produce a smaller result.
This being said, we can compare the different distributions of shares.
Let us start with the proof of equation (15) when $\gamma > 2$:
We compare equation (3) with $\underline{\boldsymbol{\lambda}}$ and $\tilde{\boldsymbol{\lambda}}$:

$$\lambda_1\gamma\big[n-\big((n-x)\lambda_1^{\frac{1}{\gamma-1}}+x(\tfrac{1-(n-x)\lambda_1}{x})^{\frac{1}{\gamma-1}}\big)\big]-(1-\lambda_1^{\frac{\gamma}{\gamma-1}}) >$$
$$\lambda_1\gamma\big[n-\big((n-x-1)\lambda_1^{\frac{1}{\gamma-1}}+(\lambda_1+\varepsilon)^{\frac{1}{\gamma-1}}+x(\tfrac{1-(n-x)\lambda_1-\varepsilon}{x})^{\frac{1}{\gamma-1}}\big)\big]-(1-\lambda_1^{\frac{\gamma}{\gamma-1}})$$

It simplifies to:

$$(\lambda_1+\varepsilon)^{\frac{1}{\gamma-1}}+x\big(\frac{1-(n-x)\lambda_1-\varepsilon}{x}\big)^{\frac{1}{\gamma-1}}>\lambda_1^{\frac{1}{\gamma-1}}+x\big(\frac{1-(n-x)\lambda_1}{x}\big)^{\frac{1}{\gamma-1}}$$

As $\lambda_1<\lambda_2<\ldots<\lambda_n \Rightarrow \lambda_1<\frac{1}{n}$ we get:

$$\frac{1-(n-x)\lambda_1}{x}>\frac{1-(n-x)\lambda_1-\varepsilon}{x}>\lambda_1$$

This point and the concavity of the function $()^{\frac{1}{\gamma-1}}$ when $\gamma>2$ allow us to state that, if $x=1$, $\underline{\lambda}_1<\tilde{\lambda}_1$. If $\varepsilon$ grows and becomes $=\frac{1-(n-2x)\lambda_1}{x+1}$,

$$\lambda_1\gamma[n-((n-x-1)\lambda_1^{\frac{1}{\gamma-1}}+(\lambda_1+\varepsilon)^{\frac{1}{\gamma-1}}+x(\tfrac{1-(n-x)\lambda_1-\varepsilon}{x})^{\frac{1}{\gamma-1}})]-(1-\lambda_1^{\frac{\gamma}{\gamma-1}}) =$$
$$\lambda_1\gamma[n-\big((n-x-1)\lambda_1^{\frac{1}{\gamma-1}}+(x+1)(\tfrac{1-(n-(x+1))\lambda_1}{x+1})^{\frac{1}{\gamma-1}}]-(1-\lambda_1^{\frac{\gamma}{\gamma-1}})$$

The same reasoning shows therefore that $\underline{\lambda}_1<\tilde{\lambda}_1<\hat{\lambda}_1$. Then applying the same reasoning with $x=[2,\ldots,n-1]$ gives our result when $\gamma>2$. The reverse is true when $\gamma<2$ by convexity of the function $()^{\frac{1}{\gamma-1}}$.
As to the trivial case of $\gamma=2$, it is simply the root of a quadratic function. ■

**Proof of Proposition 2**
To prove Proposition **??**, it is enough to prove that, for everyone, the payoff from generalized cooperation after redistribution is greater than the payoff before redistribution.

Let us first assume that only one player has a share, $\lambda_n<1$, such that $\pi_i^C>\pi_i^N$, all the $n$ players are therefore producing the deviation level of effort. As we know that, in the case of equality of shares, nobody wants to deviate from the cooperative equilibrium, we are done if we can prove that, even for the richest player, the profit of generalized cooperation is greater than the Nash payoff.

$$\frac{1}{n}n - \gamma^{-1} > \lambda_n \left( \sum_i \lambda_i^{\frac{1}{\gamma-1}} \right) - \gamma^{-1} \lambda_n^{\frac{\gamma}{\gamma-1}}$$

After rearranging, we get for $\gamma = 2$:

$$\lambda_n^2 - 2\lambda_n + 1 > 0$$

which is always true.

Then, let us assume that more than one player can afford to cooperate. As $\underline{\delta} = \delta_{cond1}$ when we use our renegotiation-proof and coalition-proof punishment, the condition to be satisfied ultimately is $\pi_i^C > \pi_i^N$.

Let us therefore define the minimal share compatible with sustainable cooperation for all the agents to be able to cooperate, $\lambda_{min}$ such that $\pi_i^C = \pi_i^N$. If some have too small a share, the minimal share compatible with sustainable cooperation becomes, if $\gamma = 2$, $\lambda_x$ such that

$$\lambda_x = -[n - (n_D + 1) + \sum_D \lambda_D] + \sqrt{[n - (n_D + 1) + \sum_D \lambda_D]^2 + 1}.$$

The distribution is in this case $\lambda_1 < \ldots < \lambda_D < \lambda_x < \ldots < \lambda_n$ where agents from 1 to $D$ can not afford to cooperate. $\boldsymbol{\varepsilon} = \sum_i \varepsilon_i = \sum_{i=1}^{D} \lambda_{min} - \lambda_i > 0$ and $\beta_i = \frac{\lambda_i}{1 - \sum_D \lambda_D}$.
We are going to show that if the wealth to transfer is supported by the cooperative players according to their share, all the agents will gain from this redistribution. We therefore compare the cooperation profit with $n$ players after redistribution to the cooperation profit with $n_C$ players with no redistribution. It boils down to verify that:

$$(\lambda_x - \beta_x \boldsymbol{\varepsilon})n - \gamma^{-1} > \lambda_x(n - n_D + \sum_D \lambda_D) - \gamma^{-1}$$

Note that, as we use a proportional rule of redistribution, it is exactly the same inequality for each player belonging to $(x, \ldots, n)$. After replacement and simplification, we get:

$$\begin{array}{rcl} (1 - \sum_D \lambda_D)(n_D - \sum_D \lambda_D) & > & \varepsilon n \\ (1 - \sum_D \lambda_D)(n_D - \sum_D \lambda_D) & > & n \sum_{i=1}^{D} \sqrt{(n-1)^2 + 1} - (n-1) - \lambda_i \end{array}$$

It becomes:

$$n_D[n(n-1) + 1] + \sum_D \lambda_D(n + \sum_D \lambda_D - 1 - n_D) > n n_D \sqrt{(n-1)^2 + 1}$$

As we know that the second term of the lefthand side of the inequality is positive - given $n \geq n_D + 1$ -, we are done if we can prove that

$$n_D[n(n-1)+1] > nn_D\sqrt{(n-1)^2+1}$$

It is so as $(n-1)^2 > 0$. ■

## References


[1] Abreu, D., 1986. Extremal Equilibria of Oligopolistic Supergames. Journal of Economic Theory 39, pp 191–228.

[2] Abreu, D., 1988. On the Theory of Infinitely Repeated Games with Discounting. Econometrica 56(2), pp 383–396.

[3] Baland, J.-M., 2005. 'Inequality and collective action on the commons', in : *Environment, Inequalities and Collective Action*, edited by Marcello Basili, Maurizio Franzini and Alessandro Vercelli, Routledge Inc.

[4] Baland, J.-M., and Platteau, J.-P., 2003. 'Economics of common property management regimes' in *Handbook of Environmental Economics*, vol. I, edited by K. G. Mahler and J. Vincent, Elsevier, North Holland, Chapter 4, pp 129-90.

[5] Banerjee, A., Mookherjee, D., Munshi, K., and D. Ray, 2001. Inequality, control rights and rent seeking: Sugar cooperatives in Maharashtra. Journal of Political Economy 109(1), pp 138190.

[6] Banerjee, A., Iyer, L., and Somanathan, R., 2006. 'Public Action for Public Goods', forthcoming in *Handbook of Agricultural Economics*, vol. IV, edited by T. Paul Schultz, Elsevier, North Holland.

[7] Bardhan, P., 2000. Irrigation and Cooperation: an Empirical Analysis of 48 Irrigation Communities in South India. Economic Development and Cultural Change 48, pp 847–865.

[8] Bardhan, P., Singh, N., 2005. Inequality, Coalitions and Collective Action, mimeo, University of California at Berkeley.

[9] Bardhan, P., Ghatak, M., and A. Karaivanov, 2006. Wealth Inequality and Collective Action. Forthcoming in the Journal of Public Economics.

[10] Bergstrom, T., Blume, L., Varian, H., 1986. On the private provision of public goods. Journal of Public Economics 29, pp 25–49.

[11] Bernheim, D., Peleg, B., Whinston, M., 1987. Coalition-Proof Nash Equilibria I. Concepts. Journal of Economic Theory 42, pp 1–12.

[12] Bernheim, D., Ray, D., 1989. Collective Dynamic Consistency in Repeated Games. Games and Economic Behavior 1, pp 295–326.

[13] van Damme, E., 1989. Renegotiation-proof equilibria in repeated Prisoners' Dilemma. Journal of Economic Theory 47, pp 206–217.

[14] Dayton-Johnson, J., 2000. The Determinants of Collective Action on the Local Commons: A Model with Evidence from Mexico. Journal of Development Economics 62, pp 181–208.

[15] Farrell, J., Maskin, E., 1989. Renegotiation in Repeated Games. Games and Economic Behavior 1, pp 327–360.

[16] Friedman, J., 1971. A noncooperative equilibrium for supergames. Review of Economic Studies 38, pp 1–12.

[17] Fudenberg, D., Tirole, J., 1991. *Game Theory*, The MIT Press.

[18] Itaya, J.-I., and Yamada, N., 2003. Private provision of public goods in renegotiation-proof equilibria, mimeo.

[19] Kandori, M., 1992. Social Norms and Community Enforcement. Review of Economic Studies 59, pp 63–80.

[20] Khwaja, A., 2006. Can good projects succeed in bad communities?, mimeo, Harvard University.

[21] Olson, M., 1965. *The Logic of Collective Action: Public Goods and the Theory of Groups*, Cambridge, MA: Harvard University Press.

[22] Ray, D., Baland, J.-M., Dagnelie, O., 2007. Inequality and Inefficiency in Joint Projects. Economic Journal 117 (July), pp 922–935.

[23] Tarui, N., 2007. Inequality and Outside Options in Common-Property Resource Use. Journal of Development Economics 83, pp 214–239.

[24] Vasconcelos, H., 2005. Tacit Collusion, Cost Asymmetries, and Mergers. RAND Journal of Economics 36(1), pp 39–62.