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The social structure of trust

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Abstract

This paper addresses the way in which the level of trust in cooperative relations depends on network structures. The study is an elaboration of models developed in earlier papers using iterated games [Weesie, J., Buskens, V., Raub, W., 1998. The management of trust relations via institutional and structural embeddedness. Journal of Mathematical Sociology (in press); Buskens, V., 1995. Social Networks and the Effect of Reputation on Cooperation. ISCORE Paper No. 42, Utrecht University]. We distinguish individual and group network measures. Applying a combination of approximation methods to the game-theoretic solution to the model, we derive hypotheses on the effects of density, outdegree centrality, and centralization on the level of trust a trustor can have in a trustee. We conclude that higher density and outdegree induce more trust. Centralization increases trust if it is 'well organized,' i.e., actors who can place more trust are central in the network. Furthermore, we discuss theoretical evidence that the relative importance of density compared to outdegree increases if the trust problem at the dyadic level is large. Finally, we show that, in many situations, a few simple network measures explain most of the effects of the network structure as a whole. © 1998 Elsevier Science B.V. All rights reserved.

1. Introduction

In many situations, social and economic exchange presupposes trust. The one who buys a used car has to trust the seller when he quotes a 'fair price' for his car, because the buyer does not have the ability to judge the quality of the car accurately. The man who lends money to his friend believes that this friend will not run away with the money, and will pay him back as soon as possible. For more examples of trust situations, We refer the reader to Snijders (1996). Incentives for opportunistic behavior by the trustee or potential damage for the trustor can be reduced or eliminated by

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contractual agreements or ex ante commitments, for example. However, complete contracts are expensive and in most cases impossible. Therefore, 'trust with reason' can have a positive influence on the efficient arrangement of transactions. For reasons of clarity, we will refer to the 'trustor' as the buyer and to the 'trustee' as the seller in this paper.

Problematic social situations such as those described above have been formalized and described as games with two players and two periods of play. These games have been referred to as Trust Games (Dasgupta, 1988; Kreps, 1990a,b). A Trust Game is a 'one-sided Prisoner's Dilemma Game.' First, the buyer chooses whether or not he trusts the seller. If he does not place trust, the game is over and both players receive a payoff P. If he places trust, the seller decides whether to honor or to abuse this trust. If the seller honors trust, both players receive $R > P$. However, if the seller abuses trust, the seller receives $T > R$ and the buyer receives $S < P$. This implies that it is better for the buyer not to place trust if he has no reason to believe that the seller will honor trust. While in the one-shot situation not placing trust is the equilibrium solution, game-theoretic modeling has shown that where there are repeated interactions the cooperative solution (placing and honoring trust) can be the equilibrium (Dasgupta, 1988; Kreps, 1990a,b; Lahno, 1995a,b). These studies focus on two players who play the Trust Game a number of times, which gives the buyer the possibility of punishing any eventual abuse of trust in subsequent games by withdrawing his trust. On the other hand, the seller has the possibility of building up a reputation of being trustworthy during a game. Essential in these studies is that the buyer can use the information he obtains about the behavior of the seller later on in the game.

This situation can be made more complicated by introducing third parties, in particular, more buyers. The effect of reputation also becomes more complicated then. A buyer can now receive information on the seller's behavior from his own interactions with the seller and from other buyers. Raub and Weesie (1990) show in a somewhat different setting that this kind of 'network embeddedness' reduces the restrictiveness of the social conditions under which problematic social situations have to be solved. Burt and Knez (1995a,b) provide empirical evidence for the importance of third parties in social interactions. One of their findings is that third parties support trust as well as distrust. The theoretical explanation they offer is that the principal effect of information from third parties is to reduce uncertainty about the behavior of the seller. This strengthens trust as well as distrust.

From the studies, we learn that the trust problem in situations that resemble Trust Games can be solved with at least two network effects. First, a buyer obtains information about a seller's earlier transactions. Second, a buyer has the possibility of informing potential buyers that a seller is abusing trust. The buyer trusts the seller because he got a good product for a fair price on an earlier occasion or because the buyer has received positive information from another buyer who had bought something from the same seller. On the other hand, if a seller sells a bad product to a buyer with whom he expects to carry out more transactions in the future or who will divulge details of this unsatisfactory transaction to many other buyers, the seller will 'spoil' many future interactions. Both arguments emphasize the role of information transmitted through social ties. We refer to the possibility of obtaining or spreading information about a seller's trustworthiness as reputation effects. In this paper, we analyze in more detail the origin of these reputation effects as a function of network position and specific network structure. In particular, we try to find a theoretical answer to the following question: In which way does a buyer's level of trust in a seller depend on his 'local' network position and on the global network structure?

To illustrate this question, consider the networks shown in Fig. 1. In each network, we distinguish six buyers numbered 1 to 6. Arcs indicate which buyer talks to which other buyer. In the illustration, although it is not essential for the model, we assume that relations are symmetric. Network (a) and network (b) are homogeneous networks, i.e., all buyers talk to the same number of other buyers. The difference between network (a) and (b) is the density, i.e, the proportion of ties present in the network. One expects that if density is higher, information will spread faster through the network and the reputation effect will be larger, which increases the possible level of trust. In this paper we concentrate on network aspects that go beyond the density effect. Network (c) and (d) have the same density as network (b), but the structure is different. In network (c), we observe individual differences. Buyers 2 and 4 talk to four other buyers. Buyers 1 and 6 talk to three other buyers, and buyers 3 and 5 only talk to two other buyers. Again,

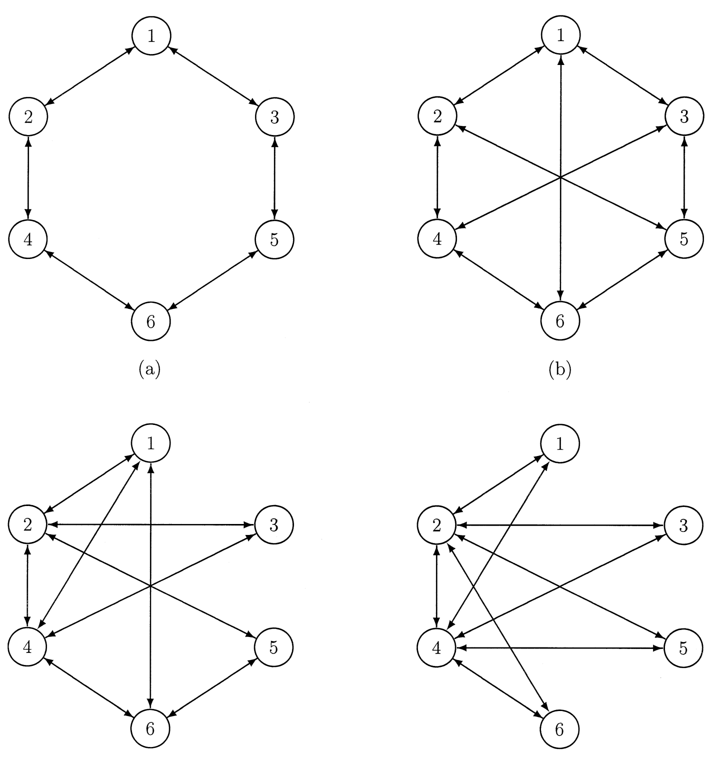

Fig. 1. Four illustrative networks.

an intuitive reasoning about the reputation effect of the number of ties of a buyer is straightforward: buyers who talk to more others, or have a higher outdegree, can spread information more easily and, therefore, will have a larger reputation effect. Still, there are other structural aspects that could have an influence and where the intuitive reasoning is less straightforward, for example: Is there an effect from the number of contacts of the buyers a certain buyer talks to? And does this effect exist even above density and outdegree effect? What is the effect of the network centralization we see in network (d) on the level of trust?

To answer these questions, we have to expand the implications of the models developed in earlier papers quite considerably (Buskens, 1995; Weesie et al., 1998). The game-theoretic model discussed by Buskens (1995) has a unique payoff dominant equilibrium in iterated trust situations with heterogeneous networks. Furthermore, conditions were given for optimal networks. The network result that emerged was a monotonic effect in the strength of ties: if the strength of ties between two types of buyers increases, the possibilities for placing trust increase. The mechanism causing this phenomenon can be described in rough terms in the following way: the level of trust increases if the information about the behavior of the seller spreads rapidly through the network and, consequently, the reputation effect of any defective behavior of the seller is larger. We will elaborate on the relationship of this paper with the literature on diffusion of information in Section 2. However, the mechanism does not imply that if we compare two networks with different densities, the level of trust will be higher for all buyers in the network with higher density. We can only anticipate that this will be the case for most buyers in most networks. This paper makes this intuition more precise and extends the results to other network measures.

We use two approximation methods to extend the analytically obtained results and to uncover the implications of the model concerning network structures. The first method is a linearization of the equilibrium of Buskens (1995, forthcoming) for homogeneous networks. This analysis shows that density and outdegree are the most important network measures, at least ‘in the neighborhood’ of homogeneous networks. Since the model is incorporated in a game-theoretic context that includes other variables describing the properties of the trust problem such as incentives for the seller to abuse trust, we find indications for the relative importance of outdegree and density dependent on these other variables. To verify that these local results also hold in networks that are not ‘close’ to homogeneous networks we need the second approximation method. This method is a simulation analysis similar to Yamaguchi (1994). For arbitrary networks the predictions of the model for the level of trust of the buyers in the seller are calculated and regressed on the network measures.

This paper is organized as follows. Section 2 reviews some important results from the literature and provides intuitive ideas about the effects of some well known network measures. Section 3 describes the important aspects of the model and briefly summarizes the analytic results derived by Buskens (1995, forthcoming). Section 4 discusses the definition of different network measures for stochastic block models. In Section 5, we linearize the complicated equilibrium conditions to obtain insight into the importance of some network measures. The computer simulation, generating predictions for large numbers of networks, is discussed in Section 6. Section 7 summarizes the implications of the two approximation methods and discusses some of the advantages and limitations of the methods and the model.

2. Established findings and intuitive ideas

The influence of social structure on human behavior is one of the main areas of research in social network analyses. Two kinds of questions are addressed. First, in network exchange theory the *distribution* of resources between actors is studied as a function of the network structure that describes opportunities for exchange in pairs. The actor who obtains the largest profit is considered to be the most powerful actor in the network. Theories that predict power have been developed by researchers such as Cook et al. (1983), Markowsky et al. (1988), Friedkin (1992), Bienenstock and Bonacich (1992, 1993), and Yamaguchi (1996). Although the methods such as game-theoretic modeling used in these papers are sometimes close to our methods, the questions addressed are rather different.

In the second type of network research, however, the questions and findings are more closely related to this paper. Social networks are now considered to offer opportunities that are instrumental to providing an *efficient* solution to a certain problem. An example is information diffusion research, which tries to predict the rate at which information spreads through a social network. This research has applications in many different fields including disease transmission (Bailey, 1975; Altmann, 1993), diffusion of innovations (Coleman et al., 1966; Burt, 1987; Strang, 1991), and information flow through networks (Rapoport, 1953a,b; Friedkin, 1982; Freeman et al., 1991; Yamaguchi, 1994). The relationship with questions posed in this paper is straightforward. If information about opportunistic behavior of the seller spreads rapidly through a network, the seller is likely to behave less opportunistically because he is afraid of losing a good reputation. While in almost all of the above-mentioned studies the network is the only variable explaining the actor's behavior being studied, we explicitly try to explain how a trust problem can be solved with help of the properties of the trust problem—such as the incentive to abuse trust—in combination with the network structure.

In the literature, we already find several results that indicate how the rate of information diffusion depends on network measures. Some of these results are even directly related to trust. The most established result is that the level of trust that can be placed is higher if the network has a higher density (see, for example, Coleman, 1990, Chap. 5 Granovetter, 1985). A classical example is given by Wechsberg (1966), who describes the diamond dealers in the diamond district of London. In this close community, diamonds are handed from one to another for inspection prior to sale without written specifications of such transactions.

A second finding is that central actors in a network adopt innovations faster and can be expected to receive and spread more information about innovations (see, for example, Coleman et al., 1966, p. 85; Rogers, 1995, pp. 273–274). The most straightforward way to define the centrality of an actor is to count the number of ties he has, known as the degree or degree centrality of an actor. In the case of directed ties, outdegree (ties leaving an actor) and indegree (ties going to an actor) can be distinguished. In terms of our problem, we state that a higher indegree implies that a buyer receives more information about the behavior of the seller and a higher outdegree implies that the buyer can inform a larger number of buyers about the behavior of the seller. Therefore, we expect that both an increase in the indegree and an increase in the outdegree increases the level of trust a buyer can have in the seller.

These findings can be generalized by arguing that the level of a buyer's trust is higher if he talks to buyers who talk to many other buyers, because the message will spread at a higher rate through the network assuming that people keep on spreading the information. Similarly, we argue that the amount of information received is larger if a buyer listens especially to buyers with a higher indegree and, therefore, that listening to buyers with a higher indegree increases the level of trust.

A third finding concerning the spread of information is the assertion of the *strength of weak ties* by Granovetter (1973). In our case, this assertion can be illustrated with help of two closely connected communities of buyers, who all trade with the same seller. In such a situation, the ties between the two communities are essential if information about deceit by the seller is to be spread through both communities. These so-called 'bridges' can have a relatively large influence on the level of trust that buyers in different parts of the network can have in the seller. Still, the diffusion of information may be hindered by concentration of ties locally that can cause information to be caught in one part of a network. This can be indicated by a local density measure (Yamaguchi, 1994), but also by the transitivity of a network. Another reason why the transitivity can be expected to have a negative effect on the level of trust buyers experience is that in transitive networks more of a buyer's contacts are also contacts of these contacts themselves and this causes more redundant information transfer.

Finally, we want to pay some attention to the centralization of the network. Intuition leads us here in different directions. If there is considerable variation in outdegrees and indegrees, but actors who obtain a great deal of information do not pass it on to many others, information diffusion may not be very efficient. On the other hand, if actors with high indegrees have also high outdegrees, information diffusion may be more efficient than in homogeneous situations.

Note that the network measures in this section can be divided in global network measures and individual network measures. The global network measures are properties of the network as a whole: density, transitivity, and centralization. The individual network measures are properties of one actor in the network: outdegree and indegree centrality. The dependent variable, namely, the level of trust a buyer can have in the seller, however, is always at the individual level. Of course, individual trust can be influenced by global network properties.

This section has shown that the literature already provides a considerable amount of findings about the effect of network measures on different dependent variables. The remainder of this paper tries to deduce formal arguments for most of the findings. Additionally, the model developed enables us to hypothesize about the network measures for which arguments are less straightforward. Finally, because we combine transactions between buyers and sellers and network measures for the network of buyers, we obtain hypotheses about interaction effects between the properties of the transactions and the network measures.

3. The model

The model that is analyzed in this paper is discussed in greater detail by Buskens (1995, forthcoming). [1] Here we confine ourselves to summarizing the important elements. A seller is involved in transactions with a large number of buyers. These transactions are modeled as Heterogeneous Trust Games (HTG). [2] The HTG is an adapted version of the ordinary Trust Game (Dasgupta, 1988; Kreps, 1990a,b). If the buyer does not place trust, both players receive the punish payoff P, without any further moves. If the buyer places trust and the seller honors trust (the cooperative solution), both players receive the reward payoff $R > P$. The third and last possibility is that the buyer places trust and the seller abuses trust. Then, the buyer receives $S < P$ and the seller $T_\theta = R + \theta$, where the temptation to abuse trust $\theta = T_\theta - R \geq 0$ is chosen from a probability distribution, F:

$$F(\theta) = \frac{\theta}{a + \theta}, \theta \in [0, \infty), a > 0.$$

Note that a is the median of this distribution. In the standard Trust Game, the seller receives a fixed payoff $T > R$ after abusing trust. Heterogeneity, thus, refers to the variation in the incentive $\theta = T_\theta - R$ of abusing trust. In a one-shot Trust Game, the game-theoretic solution prescribes that the buyer does not place trust since the seller has an incentive to abuse trust and this abuse cannot be punished. However, in an iterated Trust Game, the short-term incentive to abuse trust may be compensated by the possibility for the buyers to punish the seller after an abuse of trust (see Friedman, 1971; Axelrod, 1984). The iterated standard Trust Game has usually two possible outcomes depending on the properties of the game, that is to say, trust can always be placed or trust can never be placed. The advantage of the Iterated Heterogeneous Trust Game (IHTG) is that the possibility of placing trust for the buyer depends on the incentive to abuse trust, θ. The solution to the game will result in a threshold value, ϑ, for each buyer and buyers place trust if and only if $\theta \leq \vartheta$. We call the threshold ϑ, the level of trust the buyer can place in the seller.

In our scenario, one seller has more than one interaction with one buyer. In addition, the seller has future interactions with *other* buyers, while the different buyers can exchange information about the behavior of the seller. More specifically, we distinguish k types of buyers. The fraction of buyers of type i in the total population equals π_i, $\pi = (\pi_1, \ldots, \pi_k)$. All these buyers have transactions with the seller. Among the buyers there exists a network structure that is represented by a stochastic block model in a matrix $\mathbf{A}$ in which α_{ij} represents the density of ties between buyers of type i and j. In other words, α_{ij} is the probability that a tie exists between a specific buyer of type i and another buyer of type j. Note that α_{ii} is the density of ties within the group buyers of

[1] All derivations of formulas and results of this section can be found in the work of Buskens (1995).

[2] Note that the term heterogeneous is used in two senses in this paper. First, for a network in which the density of ties is not everywhere the same and, second, for the Trust Game with an incentive to abuse trust that is not always the same.

type i and is not necessarily equal to 1. Thus, the network **A** represents the communication structure among the buyers. The seller is a separate actor with whom the buyers have transactions.

After a buyer of type i and the seller have had a transaction, two things might happen. First, the same buyer has another transaction with the seller. This happens with a probability $1-\delta$. The parameter δ is called the 'death' rate.[3] Second, with probability δ the buyer involved dies and a new buyer has to be chosen. A buyer j is chosen with probability π_j.[4] Before the new buyer has a transaction with the seller, the old buyer has the possibility of telling the new buyer how the seller behaved in former transactions. If two buyers have the possibility of exchanging information, they will always exchange accurate information. Between a buyer of type i and type j, this happens with a probability α_{ij}. If information is exchanged, the new buyer takes this information into account in his subsequent decisions. In this way, a buyer who is deceived by the seller or who has heard from another buyer that this buyer was deceived has the possibility of passing this information onto other buyers. This offers more punishment possibilities for the buyers compared to the situation in which only mutual punishment is possible. If the old buyer does not exchange information with the new buyer, the information known to the buyer is lost. The old buyer will not have a new opportunity to exchange information. Thus, the buyers act sequentially, that is, during a series of transactions carried out by one buyer, the seller does not have interactions with other buyers and buyers can only exchange information at the time a new buyer succeeds an old one.[5]

In the game described above, the buyers are not competitors and this means that the game one buyer plays with the seller does not influence games with other buyers. Therefore, the buyers do not have an incentive to give other buyers false information. On the contrary, all buyers profit if there is honest and as extensive as possible information transfer between them. Furthermore, the strategies studied below prescribe that any buyer who has information about any deceit committed by the seller should punish the seller by not placing trust in him although the deceit might not have been omitted against himself and placing trust does not correspond to the buyer's self-interest. These and other considerations with respect to models similar to the one described here are mentioned by Williamson (1996, pp. 153–155), for example. Still, to what extent the less plausible assumptions lead to undesirable outcomes in the model, especially with respect to effects of individual network positions on trust, is subject to further research.

[3] Types of buyers can, in principle, differ in their death rate and in other parameters of the model, such as the payoffs in the HTG. Because we are now mainly interested in network aspects, we assume homogeneity for the parameters that are not related to the network structure.

[4] We assume that the distribution of buyers does not change because the groups of buyers are infinitely large. One could also assume that the buyer who dies is replace by a new one in the same position in the network or that the same buyer is replaced in his position without remembering the former transactions with the seller.

[5] The assumption that the buyers act sequentially is necessary to make the model tractable. More details about the assumptions, the order of transactions, and the possibilities for information transfer can be found in an earlier paper (Buskens, 1995). New papers are in preparation that try to relax some of the most problematic assumptions (see, for example, Buskens forthcoming).

In Buskens (1995, forthcoming), we studied equilibrium conditions for the IHTG in so-called trigger strategies. [6] These strategies prescribe that a buyer places trust if and only if he does not have any information about an abuse of trust by the seller and if the incentive θ for the seller to abuse trust is smaller than or equal to a threshold ϑ. For the IHTG, there exist thresholds ϑ_i for buyers of type i in the IHTG, which give equilibria in trigger strategies. The equilibrium threshold for a buyer of type i, ϑ_i, represents the maximum incentive for opportunism by the seller for which a buyer of type i 'dares' to place trust. It can be proven that there exists a unique Pareto-optimal equilibrium in these trigger strategies. This equilibrium is considered to be the solution to the game and the related trust thresholds ϑ^* are interpreted as the level of trust of a buyer of type i. The vector of these trust thresholds is implicitly defined by (the maximal solution of):

$$\vartheta^* = (R-P)\left((\mathbf{I}-\omega\mathbf{T})^{-1}-\mathbf{I}\right)F(\vartheta^*), \tag{1}$$

where R, P are the reward and punish payoff for the seller, ω is the depreciation rate for the seller, $F(\vartheta^*) = (F(\vartheta^*), \ldots, F(\vartheta^*))$, $\mathbf{T}$ is a $k \times k$ transition matrix based on $\mathbf{A}$, π, and δ, in which $\mathbf{T}_{ij}$ represents the possibility that a buyer of type j receives information of a buyer of type i after a transaction by that buyer. If the buyer places trust, the incentive to abuse trust for the seller is always less than his expected loss of interactions with buyers who know about the abuse of trust. Therefore, the seller never abuses trust in equilibrium. As stated before, the advantage of the IHTG over the iterated standard Trust Game is that we obtain a measure for the level of trust that can be placed by a certain buyer, while in the Trust Game, the solution would have been that a buyer places always trust or does never place trust.

In Buskens (1995, forthcoming), we have shown that:

- ϑ^* increases in the sanctioning potential, $R-P$, of a buyer;
- ϑ^* increases in the depreciation rate, ω, of the seller;
- ϑ^* decreases in the death rate, δ, of the seller;
- ϑ^* decreases in the seller's median temptation a for opportunistic behavior;
- ϑ^* increases in the strength α_{ij} of network ties. Moreover, if buyers of type j communicate information to buyers of type l, that is $\alpha_{jl} > 0$ (in other words, if any buyers of type j and l are connected) ϑ^* increases in α_{ij}. Thus, if all buyers are connected, the level of trust for all buyers increases if any α_{ij} increases.

These assertions are largely in agreement with our intuition. If the sanction potential of the buyers is larger, the seller faces larger long-term costs from opportunistic behavior. Hence, the seller will be more easily deterred from giving in to the short-term gain of opportunistic behavior, and so the level of trust for the buyer will be higher. If the future is more important, the seller will be more concerned with future interactions and again the long-term costs of opportunistic behavior will be larger and, therefore, the level of trust placed increases. If a buyer expects a longer life, and hence more interactions with the seller, he places more trust. If the incentives to abuse trust are less, the buyers place more trust. Finally, if there are more or stronger individual network

[6] Details with respect to trigger strategies can be found in Friedman (1971).

ties, more trust is placed. [7] If this involves a buyer directly (for example, consider a buyer of type i and α_{ij} increases), this is an outdegree effect. For the other buyers the effect is indirect and may be interpreted as a density effect.

The monotonicity of the effect of the strength in individual network ties α_{ij}, however, does not answer the question about the contribution of specific network characteristics to the level of trust buyers can place. Whilst monotonicity suggests that the effects of outdegree and density are positive, it does not imply that if we compare two networks with different densities, more trust can be placed by the buyers in the network with higher density. Furthermore, this monotonicity does not say anything about the effect of other network measures, such as transitivity. To identify these effects we have to isolate the network effects from the effects of other parameters. In Sections 5 and 6, we try to simplify Eq. (1) in such a way that the influences of network measures become more apparent. In Section 6, we predict the level of trust as a linear combination of network measures.

4. Network measures for stochastic blockmodels

4.1. Introduction

In this section, a number of measures are discussed. We distinguish the network measures by their aggregation level. First, we consider measures at the *individual level* of the buyers: outdegree, indegree, and other forms of centrality. Second, we consider measures at the *group level*: density, centralization measures, and transitivity. A large number of centrality and centralization measures have been proposed (for example by Bonacich, 1972, 1987; Freeman, 1979; Snijders, 1981; Stephenson and Zelen, 1989; Freeman et al., 1991; Friedkin, 1991). These include outdegree, indegree, betweenness, closeness, information, and flow-betweenness centrality and centralization.

The network model that is incorporated in the game-theoretic model corresponds to a stochastic blockmodel (Wasserman and Faust, 1992, p. 695). All random dyadic variables α_{ij}, pertaining to the percentage of ties that exists from buyers of type i to buyers of type j, are independent and all buyers of one type have exactly the same probability distribution for the different dyadic ties. Although there exists an extensive literature about the relation between stochastic blockmodels and network data (see, for example, Holland et al., 1983; Wasserman and Anderson, 1987; Anderson et al., 1992), we do not know any publication that treats network measures for stochastic blockmodels systematically. Wasserman and Faust (1992) indicate a natural way of defining these measures, namely, by taking the expected value for a randomly chosen network that is represented by the stochastic blockmodel. We focus in this paper on outdegree and indegree centrality and centralization and some additional, directly related measures that are straightforwardly to derive for stochastic blockmodels. For example, the outdegree of a buyer of type i is the expected proportion of ties from this buyer to all other buyers.

[7] Note that the α_{ij} in the blockmodel can be interpreted as the percentage of ties that exists between buyers of type i and j, or as the average frequency of contacts between these buyers.

The expected proportion to buyers of type j equals α_{ij} while the proportion of buyers of type j in the whole population equals π_j. Therefore, if we sum the product $\pi_j \alpha_{ij}$ over j, we obtain the outdegree for buyers of type i.

4.2. Individual network measures

4.2.1. Outdegree (degree centrality)

The outdegree is defined as the number of ties from an actor to the other actors and in standardized form it is the proportion of ties to other actors. We define a standardized measure for stochastic blockmodels by the expected fraction of ties that exists from a certain actor to the other actors (see the example given above). The outdegree for buyers of type i equals:

$$D_{\text{out}}(i) = \sum_{j=1}^{k} \pi_j \alpha_{ij} = (\mathbf{A}\boldsymbol{\pi})_i. \tag{2}$$

Outdegree measures the proportion of other buyers who can be reached ‘in one step.’ The network position of these alters does not affect the outdegree. Therefore, the outdegree is strictly local. Still, it may be expected that information transmitted to buyers who also have better network positions will have more impact than information given to peripheral alters. For this reason, we considered the expected proportion of buyers who can be reached in two, three, or more steps. These will be called higher order outdegrees or 2-outdegree, 3-outdegree and so on. The n-outdegree of a buyer of type i is defined as:

$$D_{\text{out}}^{n}(i) = \left((\mathbf{A}\mathbf{D}_{\pi})^{n}\mathbf{1}\right)_i, \tag{3}$$

where $\mathbf{D}_{\pi}$ is the diagonal matrix with the distribution of buyers on the diagonal. [8]

4.2.2. Indegree (degree prestige)

The indegree is the number or proportion of incoming ties from other actors. It is also called degree prestige. For stochastic blockmodels the indegree, analogously to outdegree, can be defined as.

$$D_{\text{in}}(i) = \sum_{j=1}^{k} \pi_j \alpha_{ji} = (\boldsymbol{\pi}'\mathbf{A})_i. \tag{4}$$

The n-indegree can be defined analogously to the n-outdegree:

$$D_{\text{in}}^{n}(i) = \left(\mathbf{1}'(\mathbf{D}_{\pi}\mathbf{A})^{n}\right)_i. \tag{5}$$

4.2.3. Individual centralization

The outdegree and indegree are both strictly local measures of network position. We already tried to relax this by introducing higher order outdegrees and indegrees.

[8] The Bonacich centrality measure (Bonacich, 1987) with parameters α and β is equivalent to $\alpha D_{\text{out}} + \sum_{n=1}^{\infty} \alpha \beta^{n} D_{\text{out}}^{n+1}$.

However, it is questionable whether these higher order degrees provide further explanation of the effect the network position can have on the level of trust buyers might have in the seller, because higher order outdegrees are often highly correlated with the first order outdegree. If you have a low outdegree, your higher order outdegrees are unlikely to become very high. The same holds for indegrees. Other centrality measures, like closeness, betweenness, and information centrality include the whole network structure to a very large extent. Unfortunately, these centrality measures are not easy to calculate analytically for stochastic blockmodels. One could try to estimate expected values of these measures using Monte Carlo methods. Even then dealing with disconnected networks presents problems.

We propose two measures that we call *individual outdegree centralization* $C_{\text{out}}(i)$ and *individual indegree centralization* $C_{\text{in}}(i)$, which are operationalized as the covariance between the own probability of a tie to or from a certain block of actors and the outdegree or indegree of that block of buyers. Thus:

$$C_{\text{out}}(i) = \sum_{j=1}^{k} \pi_j \big(\alpha_{ij} - D_{\text{out}}(i) \big) \big(D_{\text{out}}(j) - \Delta \big), \tag{6}$$

and:

$$C_{\text{in}}(i) = \sum_{j=1}^{k} \pi_j \big(\alpha_{ji} - D_{\text{in}}(i) \big) \big(D_{\text{in}}(j) - \Delta \big), \tag{7}$$

where Δ is the density, which will be formalized in the Section 4.3. [9]

Thus, a type of buyers has a high individual outdegree centrality if the outgoing ties of this type of buyers are relatively often toward buyers with high outdegrees. Rogers (1995, p. 289) found that actors search information from opinion leaders or high status people who are expected to have higher outdegrees or indegrees. A positive effect of the individual centralization measures on the diffusion of information could indicate that these people chose a‘rational’ strategy.

4.3. Group network measures

4.3.1. Density

In discrete non-random graphs, the density of a network is defined as the number of ties divided by the possible number of ties. In stochastic blockmodel this corresponds to the expected proportion of ties present in the network. This is equal to the weighted mean of all indegrees as well as all outdegrees:

$$\Delta = \sum_{i,j=1}^{k} \pi_i \pi_j \alpha_{ij} = \sum_{i=1}^{k} \pi_i D_{\text{out}}(i) = \sum_{i=1}^{k} \pi_i D_{\text{in}}(i) = \boldsymbol{\pi}' \mathbf{A} \boldsymbol{\pi}. \tag{8}$$

4.3.2. Centralization

Centralization measures are always related to individual centrality measures. Since we used degree centrality before, we discuss centralization measures based on indegree

[9] Note that the average of α_{ij} for all j equals the outdegree $D_{\text{out}}(i)$ and the average of the outdegrees equals the density.

and outdegree. Centralization of a whole network accounts for the variation in outdegrees or indegrees of the actors in a network. A suitable measure to account for these differences is the variance of the actor degree indices: degree variance (Snijders, 1981). For stochastic blockmodels, we define degree variance as the variance in the expectations of the degrees of the different blocks of actors. This can be done for the outdegree as well as the indegree. Note again that the density is the weighted average of the outdegrees. Thus, the outdegree variance is define as:

$$S^2_{\text{out}} = \sum_{i=1}^{k} \pi_i \left(D_{\text{out}}(i) - \Delta \right)^2. \tag{9}$$

And similarly the indegree variance is defined as:

$$S^2_{\text{in}} = \sum_{i=1}^{k} \pi_i \left(D_{\text{in}}(i) - \Delta \right)^2. \tag{10}$$

Buskens (1995) showed that for given outdegrees, the network structure centralized with respect to indegree around the buyers with the highest outdegree is the structure for which all buyers have the *highest* trust thresholds. Therefore, we define a third network centralization measure called *outdegree–indegree covariance* equal to:

$$S_{\text{out, in}} = \sum_{i=1}^{k} \pi_i \left(D_{\text{out}}(i) - \Delta \right)\left(D_{\text{in}}(i) - \Delta \right). \tag{11}$$

4.3.3. Transitivity

To define transitivity, we first recollect some definitions. A triad is a triple (i,j,l) of different actors in a network. A triad is called transitive if the existence of a tie from i to j and from j to l implies that there exists a tie from i to l. A triad is called intransitive if the ties from i to j and from j to k exist but the tie from i to k does not. Other triads are called vacuously transitive. A network is called transitive if all ordered triads are transitive. We define level of transitivity of a blockmodel as the expected proportion of triples that is not intransitive. The probability that a triad (i,j,l) is intransitive equals $\alpha_{ij}\alpha_{jl}(1-\alpha_{il})$. Therefore, network transitivity is defined as:

$$\text{Tr} = 1 - \sum_{i,j,l} \pi_i \pi_j \pi_l \left(\alpha_{ij}\alpha_{jl}(1-\alpha_{il}) \right). \tag{12}$$

5. Linearization

In Section 3, we characterized the equilibrium levels of trust of the buyers in the IHTG in terms of the network structure parameters $\boldsymbol{\pi}$ and $\mathbf{A}$, and the parameters of the constituent game. The implicitness of the characterization makes it hard to interpret the implied network effects. It is difficult to derive comparative statics for specific network measures from the expression. In other words, we are not able to say whether the model predicts that the level of trust decreases or increases if, for example, the indegree increases. To obtain a better insight into the expression we use approximation methods. In this section, we apply linearization around function values where explicit expressions of the equilibria are obtained easily. Define $G(\vartheta^*(\boldsymbol{x}),\boldsymbol{x})$ as the implicit function that describes ϑ^* with the other parameters $\boldsymbol{x}$ given:

$$G(\boldsymbol{\vartheta}^*(\boldsymbol{x}),\boldsymbol{x}) = \boldsymbol{\vartheta}^* - (R-P)\left((\mathbf{I}-\omega\mathbf{T})^{-1}-\mathbf{I}\right)F(\boldsymbol{\vartheta}^*). \tag{13}$$

Because $G(\boldsymbol{\vartheta}(\boldsymbol{x}),\ \boldsymbol{x})$ is a smooth function of $\boldsymbol{x}$ and $((\partial G)/(\partial\boldsymbol{\vartheta}^*))^{-1}$ exists, which will be shown in Appendix A, we can use the implicit function theorem (Dieudonné, 1960, pp. 270–273). This implies that $\boldsymbol{\vartheta}^*$ depends smoothly on $\boldsymbol{x}$, and thus:

$$\frac{\partial\boldsymbol{\vartheta}^*}{\partial\boldsymbol{x}} = \left(\frac{\partial G}{\partial\boldsymbol{\vartheta}^*}\right)^{-1}\frac{\partial G}{\partial\boldsymbol{x}}. \tag{14}$$

In this section, we linearize around a homogeneous network $\mathbf{A}_0$, in which the buyers have ties to buyers of their own type with probability α_1 and to other types of buyers with probability α_2.

$$\mathbf{A}_0 = \alpha_1\mathbf{I} + \alpha_2\mathbf{J},\ \text{where}\ \mathbf{J} = \mathbf{1}\mathbf{1}'. \tag{15}$$

In addition, we assume that all types of buyers are equally frequent, $\pi^i = 1/k$ for all i. Note that both the density and outdegrees of this network equal $\alpha_1/k+\alpha_2$. The reason to use this network is that the trust thresholds can be calculated explicitly. First, we compute the transition matrix $\mathbf{T}_0$ (see Buskens, 1995, forthcoming):

$$\mathbf{T}_0 = (1-\delta)\mathbf{I} + \frac{\delta}{k}\mathbf{A}_0 = (1-\delta)\mathbf{I} + \frac{\delta\alpha_1}{k}\mathbf{I} + \frac{\delta\alpha_2}{k}\mathbf{J} = \eta_1\mathbf{I} + \eta_2\mathbf{J}, \tag{16}$$

where $\eta_1 = 1-\delta+\delta\alpha_1/k$ and $\eta_2 = \delta\alpha_2/k$. For $\mathbf{T}_0$, we can easily derive the payoff dominant trigger equilibrium. The maximal non-negative solution of the trust thresholds is the same for all types of buyers by reasons of symmetry and equals:

$$\vartheta^*(\mathbf{A}_0) = \frac{(R-P)(\omega\eta_1 + k\omega\eta_2)}{1-\omega\eta_1-k\omega\eta_2} - a. \tag{17}$$

We deduce a first order approximation of the equilibrium for a network matrix $\mathbf{A}$ in the 'neighborhood' of $\mathbf{A}_0$. Then, Theorem 1 gives a first order approximation of the trust thresholds $\vartheta^*(\mathbf{A})$ for $\mathbf{A}$.

Theorem 1. *If $A \approx A_0 = \alpha_1\boldsymbol{I} + \alpha_2\boldsymbol{J}$, then the solution of Eq. (1) for trust threshold $\vartheta*$ satisfies:*

$$\begin{aligned}\boldsymbol{\vartheta}^*(\mathbf{A}) &\approx \boldsymbol{\vartheta}^*(\mathbf{A}_0) + \gamma_a(\mathbf{I}+\gamma_b\mathbf{J})(\mathbf{A}-\mathbf{A}_0)1\\ &= \boldsymbol{\vartheta}^*(\mathbf{A}_0) + k\gamma_a\left(\mathbf{D}_{\text{out}}(\mathbf{A}) - \mathbf{D}_{\text{out}}(\mathbf{A}_0)\right) + k^2\gamma_a\gamma_b\left(\Delta(\mathbf{A}) - \Delta(\mathbf{A}_0)\right)\mathbf{1},\end{aligned} \tag{18}$$

with:

$$\gamma_a = \frac{\delta w(R-P)F(\vartheta_0^*)}{k(1-\omega\eta_1 - k\omega\eta_2)\big(1-\omega\eta_1\big(1+f(\vartheta_0^*)(R-P)\big)\big)} > 0, \tag{19}$$

$$\gamma_b = \frac{\big(1+f(\vartheta_0^*)(R-P)\big)\omega\eta_2}{1-(\omega\eta_1 + k\omega\eta_2)\big(1+f(\vartheta_0^*)(R-P)\big)} > 0, \tag{20}$$

and:

$$\mathbf{D}_{\text{out}} \text{ the vector of all outdegrees.} \tag{21}$$

Proof. See Appendix A.

From Theorem 1, it follows immediately that $k\gamma_a$ indicates the magnitude of the change in the trust thresholds for a change in the outdegree for a certain type of buyers, while $k^2\gamma_a\gamma_b$ measures the weight of a small increase in total density of the network. Because γ_a and γ_b are both positive, we find positive effects of outdegree and density on the equilibrium thresholds ϑ^*. Furthermore, the linearization result implies that the parameters in the game-theoretic model determine how large the relative size of the effect of changes in outdegree and density will be. The parameter γ_b, or more precisely $k\gamma_b$, can be interpreted as the relative effect of density compared to outdegree. Comparative statics of γ_a and γ_b are not straightforward, because monotonicity of γ_a and γ_b is not guaranteed for most of our model parameters. However, two monotonic effects are found, namely, γ_b increases in a and decreases in $R-P$. Therefore, we predict that density becomes more important compared to outdegree if $R-P$ decreases and if a increases.

The limitation still holds that the findings based on linearization are limited to networks ‘close’ to a homogeneous situation. We cannot generalize these findings from the linearization analysis without further examination of more heterogeneous networks. To overcome this problem, we study which of the findings for rather homogeneous networks also hold for more heterogeneous networks in the simulation described in the Section 6. Furthermore, we investigate which network measures other than density and outdegree have an effect on the trust thresholds in more heterogeneous networks.

6. Simulation

6.1. The method

In this section, we use a second approximation method to study the implications of our model, namely computer simulation. The simulation is a variant of the method used by Yamaguchi (1994). The general idea is that we have a number of independent variables, namely, the network measures and probably other relevant variables. Furthermore, there are ideas about the relation between the independent variables and a dependent variable: in our case, for example, the level of trust the buyer can have in the seller is larger if his outdegree is larger. The method used is to construct a set of networks with 'enough' variation in the independent variables. For these networks we compute the game-theoretic predictions. This forms a simulated data set. Then, we analyze the simulated data using a conventional statistical model to derive the comparative statics of the independent variables on the dependent variable.

There are a number of differences between our implementation of this idea and Yamaguchi's implementation. The most essential difference is that we modeled network embeddedness in a game-theoretic context, which makes it possible to incorporate other variables at the dyadic level in the analyses, such as the temptation to abuse trust. This enables us to include interactions of network measures and variables at the dyadic level in the analyses. In Yamaguchi's model the network measures are the only independent variables. Another difference is that the dependent variable in Yamaguchi's paper is the inefficiency of information flow in a network operationalized as the (weighted) average expected time needed for information to travel from every actor to every other actor in the network, which is a dependent variable at the network level. Our dependent variable is the level of trust a buyer can have in the seller, which is a property at the individual level. Therefore, we include not only global network measures, but also individual network measures. This difference is actually not that fundamental. It depends on the research question that has to be answered which dependent variable is chosen. Yamaguchi can analyze a dependent variable comparable to the one in this paper in a way similar to the way he did with his.

A third difference is the set of networks used for simulations. Yamaguchi studied a set of networks consisting of seven actors and six to nine ties. Because we represent networks as stochastic blockmodels, we needed a different set of networks. It was a major problem to construct networks such that the network measures vary 'enough,' without creating relations between structural properties and the trust equilibria that are due to the construction method. For instance, generating every α_{ij} uniformly from the interval [0, 1] obviously results in networks with a density close to 0.5. Therefore, we developed a couple of methods to construct networks, such that we obtained acceptable variations in density, indegree, outdegree, and the different centrality and centralization measures as discussed in Section 4. [10] By introducing dummies for the construction methods in the regression analyses later on, we can check whether the results differ with the construction methods.

Together with the network structures, we varied the other variables:

- For the distribution of incentives to abuse trust, we always used $F(\theta) = (\theta)/(a + \theta)$, with a, the median of F, equal to 1, 2, or 3, each with probability $1/3$.
- We chose the costs of contracting, $R - P$, equal to an integer between 2 and 7, each with probability $1/6$.

[10] Details on the different network construction methods are provided by Buskens (1996).

- The ‘survival’ rate, $1-\delta$, that the same buyer and seller are involved in another transaction, varied uniformly on [0, 0.75] and was chosen the same for all buyers.
- The depreciation rate, ω, varied uniformly on [0.85, 0.95].
- The number of types of buyers, k, was chosen between 2 and 8 each with probability equal to $1/7$.
- The proportion of all types of buyers in the population was chosen $1/k$.

All these choices for the different parameters were done independently for 10,000 networks. Randomly one of the construction methods was chosen and randomly one buyer was chosen for whom the equilibrium was calculated. [11] This resulted in a data set with 10,000 cases. Each case represents one buyer in a different network. For each case the individual network measures of the buyer and the group network measures are computed by our simulation software. [12]

6.2. Analyses and results

In this subsection, we draw a number of implications from the theoretical model, that are partly additional to the analytically obtained results. The first simulation results correspond to the analytic results from Buskens (1995, forthcoming) and the linearization results of Section 5:

- trust increases in sanction potential of the buyer, $R-P$;
- trust decreases in the median temptation, a;
- trust increases in depreciation rate, ω;
- trust decreases in death rate, δ;
- trust increases in network density, Δ; and
- trust increases in outdegree, $D_{\text{out}}(i)$ for buyers of type i.

Although these variables explained most of the variance in a straightforward linear regression model for the prediction of the equilibrium, the relation between independent and dependent variables was definitely not linear. Eq. (1) clearly shows that the trust thresholds are *not* a linear combination of the model parameters. Therefore, we introduce a *standardized equilibrium*, ϑ^*, in order to isolate the network effects from the other parameters and to control for the payoffs, depreciation rate of the seller, and the death rate in a manner that is based on the model. We reexamine the homogeneous case where $\alpha_{ij}=\alpha$ for all i and j. Then, the trust threshold is the same for every buyer. Restricting the analyses to cases where at least some trust is possible ($\vartheta^*>0$), the threshold equals:

$$\vartheta^* = \frac{(R-P)((1-\delta(1-\alpha))\omega)}{1-(1-\delta(1-\alpha))\omega} - a > 0, \tag{22}$$

in which the network parameter α plays a ‘complicated’ role. Eq. (2) makes it possible

[11] Alternatively, we analyzed the results including all buyers in each network controlling for the clusters of buyers from the same network. The results did not differ from the results presented in Section 6.2.

[12] The Pascal code as well as an executable program are available from the author.

to control for effects of other parameters, because for the homogeneous case the network effect can be explicitly isolated. Now, we define a *standardized equilibrium*:

$$\vartheta^* = 1 - \frac{1}{\delta} + \frac{(\vartheta^* + a)}{\delta\omega(\vartheta^* + a + (R - P))}. \tag{23}$$

For the homogeneous case, we have $\vartheta^* = \alpha$, which is the network density and describes the whole network for homogeneous cases. For heterogeneous cases we will try to predict ϑ^* with a linear combination of network measures (N_k):

$$\tilde{\vartheta}^* = \sum_k \beta_k N_k + \varepsilon, \tag{24}$$

where the network measures N_k are density or outdegree, for example, and ε is a residual term that has to be small. With this transformation, the regression results turn out to be 'almost' linear. [13] We now restrict the analyses to situations in which the equilibrium trust threshold is positive. For about 10% of the cases, the equilibrium was zero. [14] Table 1 presents the regression results for five different models.

The transformation of the dependent variable proved to be useful for different reasons. First, the relation between the standardized equilibrium and network measures was linear. Second, to a large extent the transformation accounted for the effects of $R - P$, ω, a, and δ: $\tilde{\vartheta}^*$ turned out to depend mainly on the network effects. As Model 2 shows, four network measures explain almost 99% of the variance in the standardized equilibrium and outdegree, individual outdegree centralization, and outdegree variance again explained almost 90% of the variance compared to the pure density model. We are now able to investigate which hypotheses follow from our model and to what extent they are in agreement with or additional to the findings we presented in Section 2.

First, network density has a positive effect on the level of trust a buyer can have. Second, outdegree has a positive effect on trust and is, together with density, the most important explanatory variable in the model, as has already been suggested by the results of the linearization analysis. Third, indegree does not have an effect on trust. This contrasts with our arguments in Section 2. We expect that this is mainly due to the simplifying assumptions in the model and the assumptions concerning complete and perfect information availability in particular. Fourth, it is not only important to have a high outdegree, but also to be connected to the buyers who have a high outdegree (individual outdegree centralization). Furthermore, the model implies that trust increases if the indegrees are well organized, i.e., 'buyers listen to buyers who receive much information' (individual indegree centralization). Fifth, we did not find any effect of transitivity. Finally, the findings about centralization are, as expected, not straightforward. Outdegree–indegree covariance has a positive effect although it is only significant

[13] Instead of ordinary standard errors, we calculated Huber standard errors (Huber, 1967), also known as White's robust estimates or 'sandwich' estimates. Huber's formula for the estimation of standard errors solves some problems in model estimation, for example, residuals do not need to be homoscedastic.

[14] Additional analyses were done in which all observations were included, but substantial results did not change for these analyses.

Table 1
Linear regression of the standardized equilibrium level of trust ($N = 9045$)

Independent variable	Model 0	Model 1	Model 2	Model 3	Model 4
Individual network measures					
Outdegree		0.47**	0.47**	0.47**	0.75**
Indegree				−0.00	−0.00
Ind. centralization (outdegree)			1.05**	0.86**	0.84**
Ind. centralization (indegree)				0.16**	0.18**
Group network measures					
Density	1.0**	0.53**	0.53**	0.53**	0.27**
Outdegree variance			−0.47**	−0.52**	−0.55**
Indegree variance				0.01	0.00
Outdegree–indegree covariance				0.13	0.13*
Transitivity				−0.00	0.01
Other parameters and interactions					
Death rate (δ)					0.04**
Death rate × outdegree					−0.48**
Death rate × density					0.44**
Sanction ($R-P$)					−0.002**
Sanction × outdegree					0.03**
Sanction × density					−0.03**
Temptation (a)					0.004**
Temptation × outdegree					−0.07**
Temptation × density					0.07**
Constant	0.00	0.00	0.00	0.00	−0.03*
R^2	0.868	0.976	0.986	0.986	0.994
$\tilde{R}^2$ w.r.t. density model[a]	0.000	0.817	0.893	0.895	0.952

** and * represent significance at $p < 0.001$ and $p < 0.01$, respectively, based on Huber standard errors.
[a]The $\tilde{R}^2$ w.r.t. the density model measures the explained variance after controlling for density and equals $1-(1-R^2)/(1-R^2_{\text{dens}})$, where R^2_{dens} is the explained variance of the model with density as the only independent variable (Model 0).

in Model 4. The individual equivalent of this measure, the individual outdegree centralization, accounts for the largest part of the positive effect of a 'well-organized' centralization. On the other hand, the outdegree variance, which measures centralization in general, has a negative effect on trust. No effect of the indegree variance was found.

In Model 4, we added δ, $R-P$, a, and their interaction effects with density and outdegree to the model. In the linearization analysis, we found interaction effects for the interactions with a and $R-P$. Moreover, we expected that we controlled for the direct effects of δ, $R-P$, and a in the standardized equilibrium. Model 4 shows that the direct effects are indeed small although they are significant. This is also a result of the large number of cases, which decrease the significance levels. The t-values for these variables are considerably smaller than those of the interaction effects and the important network measures mentioned in Model 2.

The interpretation of the signs of the interaction effects is not straightforward, because they indicate effects on the standardized equilibrium level of trust $\tilde{\vartheta}^*$, while we are interested in the effects on the unstandardized equilibrium of trust ϑ^*. Fortunately, it can be proven with the help of partial derivatives that we can interpreted the regression coefficients as if they are regression coefficients for the unstandardized equilibrium.

The interaction effects imply that outdegree becomes more important relative to density if the possibilities of being able to trust in the dyadic relations increase. Thus, the more problematic the dyadic situation (the higher the death rate, the smaller the sanction, and the higher the temptation), the more important density and, consequently, the whole network is. Consider the death rate, for example. If the death rate equals zero, i.e., the buyer expects to have transactions with the seller forever, then the problem on the dyadic level is relatively small, because opportunistic behavior can be punished by the buyer himself in the future. The corresponding coefficients for density and outdegree are 0.27 and 0.75, which implies that outdegree is more important than density in this case. However, if the death rate equals one, i.e., the buyer expects no more transactions with the same seller in the future and the problems at the dyadic level are larger, then the coefficient for outdegree is $0.75 - 0.48 = 0.27$, while the coefficient for density is $0.27 + 0.44 = 0.71$ and we conclude that in this case the outdegree is less important than density. The interaction effects imply that the model predicts a substitution effect from ‘global’ network measures and to ‘local’ network measures if the dyadic trust problem becomes smaller.

We will now report briefly on some additional analyses. The higher order outdegrees had significant effects, but they hardly contributed to the explained variance. We did not find any effect of the network size. The outdegree of buyers of type *i* did not have a direct effect on trust that other buyers can place after controlling for density. Dummies for the construction methods did not produce effects in the linear regression. Nevertheless, we cannot exclude that the residuals depended on the construction method. The following reasoning is important. The more homogeneous the networks the better they fit into the regression model. The model even fits perfectly for all networks where all buyers have the same outdegree. Therefore, a construction method that produces more heterogeneous networks has larger residuals than a construction method that produces more homogeneous networks. A further consequence of the fact that homogeneous networks are perfectly predicted is that the R^2 in Table 1 is not a meaningful number. If we include enough homogeneous networks, in which there is no structure and, therefore, there is nothing to explain, R^2 will approximate 1 as closely as we require. On the contrary, $\tilde{R}^2$ is a more informative number, because it indicates which proportion of the real structure is explained. [15] It turns out that if we choose a subset of heterogeneous networks for which the explanatory power of density is much lower, the $\tilde{R}^2$’s still have about the same magnitude, i.e., outdegree explains about 80% of the real structure, the other network measures another 10%, and the interaction effects yet another 5%. This is also an argument for the fact that the other network measures matter. In other words, Model 1 suggests that only density and outdegree explain virtually everything. However, if we concentrate on more heterogeneous networks, the other network measures become

[15] In fact, density describes only the amount of ties in the network. The other network measures give information about the structure of the ties.

important. Nevertheless, our set of network measures explains an important part of the variance for more heterogeneous networks as well, particularly if we include interaction effects (Model 4).

7. Conclusions and discussion

Detailed analyses showed that the model under research in this paper has substantive as well as methodological appeal. We start with the substantive implications of the model. This paper reproduces results from earlier papers (Weesie et al., 1998; Buskens, 1995). We expect higher levels of trust between partners in durable exchange relations if the sanction potential of the buyers is larger, if the seller's incentives for opportunistic behavior are smaller, if the seller's depreciation parameter is higher, if death rates for buyers are smaller, and if dyadic network ties among buyers are stronger. This paper has focused on the analysis of the effect of network embeddedness, i.e., the effect of network *structure*. By various approximation methods (linearization and simulation), we showed that the model predicts which network measures have positive or negative effects on the level of trust buyers can have in a seller. We found considerably more than only a density effect. Buyers with a higher outdegree will have higher levels of trust, even after controlling for density. Levels of trust increase if buyers direct their ties more toward buyers who have higher outdegrees. This effect was clearly shown by the individual outdegree centralization measure. A smaller positive effect was found for individual indegree centralization. The comparable group effect measured with outdegree–indegree covariance turned out to be less important. Levels of trust decrease in outdegree variance after controlling for the other measures. This implies that, while individual centralization has a positive effect on the level of trust for the buyer involved, the average level of trust of all buyers decreases in the centralization of the whole network.

In addition, several interaction effects were found analytically (in the neighborhood of homogeneous networks) and in the simulation. The most important insight from the interaction effects is that global network measures, and density in particular, become more important as the trust situation becomes more difficult, i.e., the seller has a larger incentive to abuse trust, the punishment potential for the buyer in future interactions is less, or the future is less important for the seller. For example, if the incentives for the seller to abuse trust increase, the effect of density on the level of trust becomes more important relative to outdegree. Thus, global network measures have a larger influence on the level of trust a buyer can have in a seller than local network measures if the dyadic trust situation is more problematic. More concretely, we can distinguish four situations. First, the buyer wants to buy a cheap product whose quality can easily be checked. In this case, we can hardly speak of a trust problem and the influence of the network on the trust problem will be minimal. Second, the buyer wants to buy a product that is more expensive and of more uncertain quality. Now, we expect that the direct relations of this buyer will have a particular influence on the level of trust this buyer has in the seller, while the global network structures is not important. Third, the buyer wants to buy a rather expensive and complex product, which causes a considerable trust problem. The influence of the direct relations on the level of trust decreases, while the influence of the global network structure increases, that is to say, the influence of the relatives of the direct neighbors. Therefore, in this case the whole network will have a considerable influence on the level of trust the buyer can have. Finally, it is possible that the trust problem is ‘too’ large. These are the situations where $\vartheta^* = 0$ which we did not consider in the analyses. In these situations network embeddedness cannot induce any trust for the buyer.

The model does not support our intuitions as far as the indegree is concerned. We think that this is due to the assumptions in the model that information is always and accurately passed from one buyer to another. In a forward-looking model such as we have developed, the essential basis for the buyer’s trust is the possibility of retaliation if the seller should act opportunistically. However, in the equilibrium situation trust will never be abused and information about such an abuse will never occur in the network. Therefore, it is important to have the possibility of spreading information (outdegree), but the ability to obtain the information that the seller is still trustworthy is not important. This finding can be relaxed by introducing ‘noise’ in the information transfer. For example, the buyers cannot always transfer information about the seller’s behavior accurately. In the context of noise, it is important that you can receive information from multiple sources in order to be as certain as possible that information is true. We leave this extension of the model to further research. A similar reasoning holds for the fact that we do not find an effect of transitivity.

From a methodological point of view, this paper provides a strategy for coping with the effects of network measures, and in particular for heterogeneous networks. Assuming that the network structure is given, the effect of the network can be estimated as the weighted sum of a number of easily calculable network measures: outdegree, density, individual outdegree centralization, and outdegree variance. It is notable that these four network measures describe most of the network structure, and controlling for density, the other three measures explained about 90% of the ‘structural effects.’ Therefore, given that the collection of network data is hard and costly (at least to obtain representations of complete networks), this paper presents arguments for focusing on some properties of the network. If networks are expected to be quite homogeneous, density and outdegree already explain a large part of the network effects. If networks seem to be more heterogeneous, variance and organization of outdegrees should be measured as well.

Finally, the theoretical approach used here seems promising in a wide range of network studies. Simulating a large number of representative network structures and studying the results of these structures with conventional statistical methods can increase insight into the implications of a certain model and can produce hypotheses additional to analytically obtained results (see also Yamaguchi, 1994, 1996). One of the main problems in this simulation strategy is how to obtain a representative set of networks and how to test that it actually is a representative set. A very first method used in this paper is to check the simulation results against analytic results as far as possible. If the simulation does not reproduce analytic results, you can be sure that the set of networks is not representative. Multiple methods used to generate random graphs allowed to check for ‘method effects.’ Still, the problem is not completely solved if you do reproduce the analytic results.

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Appendix A. Proof of linearization theorem

Proof. Denote $\boldsymbol{\vartheta}^*(\mathbf{A}) = \boldsymbol{\vartheta}^*$ and $\boldsymbol{\vartheta}^*(\mathbf{A}_0) = \boldsymbol{\vartheta}_0^* = (\vartheta_0^*, \dots, \vartheta_0^*)$ by homogeneity of the game in $\mathbf{A}_0$. Furthermore, we define $F(\boldsymbol{\vartheta}^*) = F(\vartheta_0^*)\mathbf{1}$, the scalar $\rho = (R - P)f(\vartheta_0^*)$, and $\tilde{\boldsymbol{T}}_0 = (\boldsymbol{I} - \omega \mathbf{T}_0)^{-1}$. By assuming that the conditions for the implicit function theorem holds, the first order approximation of Eq. (1) is:

$$\begin{aligned}\boldsymbol{\vartheta}^* &= (R-P)\left((\mathbf{I} - \omega\mathbf{T})^{-1} - \mathbf{I}\right)F(\boldsymbol{\vartheta}^*)\\ &= (R-P)\left(\tilde{\boldsymbol{T}}_0 - \mathbf{I} + \omega\tilde{\boldsymbol{T}}_0(\mathbf{T} - \mathbf{T}_0)\tilde{\boldsymbol{T}}_0\right)\left(F(\boldsymbol{\vartheta}_0^*)1 + f(\vartheta_0^*)(\boldsymbol{\vartheta}^* - \boldsymbol{\vartheta}_0^*)\right) + \varepsilon\\ &= \boldsymbol{\vartheta}_0^* + \rho\left(\tilde{\boldsymbol{T}}_0 - \boldsymbol{I}\right)(\boldsymbol{\vartheta}^* - \boldsymbol{\vartheta}_0^*) + \omega F(\vartheta_0^*)\tilde{\boldsymbol{T}}_0(\mathbf{T} - \mathbf{T}_0)\tilde{\boldsymbol{T}}_0\mathbf{1} + \varepsilon,\end{aligned} \tag{25}$$

where $\varepsilon = O(\mathbf{A} - \mathbf{A}_0)$ is small if the difference between $\mathbf{A}$ and $\mathbf{A}_0$ is small. By straightforward multiplication, we can verify the following formula $(t \neq 1/k)$ that we need below:

$$(\mathbf{I} - t\mathbf{J})^{-1} = \mathbf{I} + \frac{t}{1 - kt}\mathbf{J}, \tag{26}$$

where k is the number of types of buyers. Therefore:

$$\begin{aligned}\tilde{\boldsymbol{T}}_0 = (\mathbf{I} - \omega\mathbf{T}_0)^{-1} &= \frac{1}{1 - \omega\eta_1}\left(\mathbf{I} - \frac{\omega\eta_2}{1 - \omega\eta_1}\mathbf{J}\right)^{-1}\\ &= \frac{1}{1 - \omega\eta_1}\left(\mathbf{I} + \frac{\omega\eta_2}{1 - \omega\eta_1 - k\omega\eta_2}\mathbf{J}\right).\end{aligned} \tag{27}$$

Substituting Eq. (27) and $(\mathbf{I} + (\omega\eta_2)/(1 - \omega\eta_1 - k\omega\eta_2)\mathbf{J})\mathbf{1} = (1 - \omega\eta_1)/(1 - \omega\eta_1 - k\omega\eta_2)\mathbf{1}$ in Eq. (25) gives:

$$\begin{aligned}\boldsymbol{\vartheta}^* = \boldsymbol{\vartheta}_0^* + \rho\left(\tilde{\boldsymbol{T}}_0 - \mathbf{I}\right)(\boldsymbol{\vartheta}^* - \boldsymbol{\vartheta}_0^*) + \frac{(R-P)\omega F(\vartheta_0^*)}{(1 - \omega\eta_1)(1 - \omega\eta_1 - k\omega\eta_2)}\\ \times\left(\mathbf{I} + \frac{\omega\eta_2}{1 - \omega\eta_1 - k\omega\eta_2}\mathbf{J}\right)(\mathbf{T} - \mathbf{T}_0)\mathbf{1} + \varepsilon.\end{aligned} \tag{28}$$

Because:

$$\left(\mathbf{I}-\rho\left(\tilde{\boldsymbol{T}}_{\mathbf{0}}-\mathbf{I}\right)\right)^{-1} = \frac{1-\omega\eta_1}{1-\omega\eta_1-\rho\omega\eta_1}\left(\mathbf{I}-\frac{\rho\omega\eta_2}{(1-\omega\eta_1-\rho\omega\eta_1)(1-\omega\eta_1-k\omega\eta_2)}\mathbf{J}\right)^{-1}$$
$$= \frac{1-\omega\eta_1}{1-\omega\eta_1-\rho\omega\eta_1}\left(\mathbf{I}+\frac{\rho\omega\eta_2}{(1-\omega\eta_1)(1-(\omega\eta_1+k\omega\eta_2)(1+\rho))}\mathbf{J}\right),$$

the expressions for γ_{a} and γ_{b} follow directly from tedious but straightforward calculations. The existence of the $((\partial G)/(\partial \boldsymbol{\vartheta}^{*}))^{-1}$ is exactly guaranteed by the existence of the inverse matrix as calculated in the formula above, which ensures that we indeed could use the implicit function theorem. The expression in terms of the outdegrees and density follows immediately from the definitions.

According to the definition of the transition matrix, $\eta_1 + k\eta_2 < 1$. Because all other parts of γ_a and γ_b are clearly positive, the signs of γ_a and γ_b depend on the magnitude of $f(\vartheta_0^{*})(R-P)$. The distribution F used in the paper is concave and, therefore,

$$f(\vartheta_0^{*})(R-P) < \frac{F(\vartheta_0^{*})(R-P)}{\vartheta_0^{*}} = \frac{1-\omega\eta_1-k\omega\eta_2}{\omega\eta_1+k\omega\eta_2}, \qquad (29)$$

which is sufficient to assure that γ_a and γ_b are positive.

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