

Polarization, Fractionalization and Conflict*

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1. Introduction

A recent upsurge of empirical studies on the causes of conflict attempts to connect various features of the distribution of the relevant characteristic (typically ethnicity or religion) to conflict. The distributional indices differ (polarization, fractionalization or Lorenz-domination) and so do the various specifications of “conflict” (onset, incidence or intensity). Overall, the results are far from clear, and combined with the mixture of alternative indices and notions of “conflict” it is not surprising that the reader may come away thoroughly perplexed.

The aim of this paper is to provide a theoretical framework that permits us to distinguish between the occurrence of conflict and its severity and that clarifies the role of polarization and fractionalization in each of these cases. Our analysis brings together strands from three of our previous contributions: on polarization (Esteban and Ray, 1994, and Duclos, Esteban and Ray, 2004), on conflict and distribution (Esteban and Ray, 1999) and on the viability of political systems (Esteban and Ray, 2001).

Interest in the connections between inequality and conflict is not new, of course. Political scientists have been much concerned with these issues; see, for instance, the prominent contributions by Brockett (1992), Midlarski (1988), Muller and Seligson (1987), and Muller, Seligson and Fu (1989). Midlarski (1988) and Muller, Seligson and Fu (1989) had already voiced their reservations with respect to the standard notions of inequality as an appropriate tool for conflict analysis. To go even further back, Nagel (1974) had argued that the relationship between inequality and conflict should be non-linear. Indeed, as Lichbach’s (1989) survey concludes, the empirical studies on the relationship between inequality and conflict--and these typically posit a linear relationship--have only come up with ambiguous results.

In the area of economics, the analysis of the link between distribution and conflict was largely inspired by a desire to study pathways between inequality and growth.¹ Certainly the possibility that inequality is a determinant of social conflict and --- via this route --- impedes growth is a contender for one of the

¹ See Bénabou (1996) for a deep and comprehensive survey.

more important pathways. The most recent round of interest in this connection was triggered by the contribution of Easterly and Levine (1997) who shifted the emphasis to ethnic fractionalization rather than economic inequality, but continued to emphasize the “reduced-form” connections with growth. Among the more relevant contributions to this literature are the papers by Alesina et al. (2003), Collier (1998, 2001), Collier and Hoeffler (2004), Fearon and Laitin (2003), Hegre et al. (2001), La Porta et al. (1999), Montalvo and Reynal-Querol (2005), Reynal-Querol (2002a) and Schneider and Wiesehomeier (2006a).

But the empirical results are ambiguous, if not controversial. If the pathway between fractionalization and growth runs through conflict, it is empirically hard to spot. By and large, it is fair to say that most of the literature fails to find any significant evidence of ethnic fractionalization as a determinant of conflict. This negative finding is underlined by Montalvo and Reynal-Querol (2005) who obtain, instead, a significant relationship between ethnic *polarization* and the incidence of conflict. (As we shall see below, the two variables are often at odds with each other.) While formally not using a measure of polarization, Collier and Hoeffler (2004) also argue that the contested dominance of one large group rather than fractionalization increases the probability of civil conflict. The Montalvo-Reynal-Querol result has recently been reassessed by Schneider and Wiesehomeier (2006a) using a different data set and focusing on onset, rather than incidence, of conflict. They argue that fractionalization is a better predictor of conflict than polarization.

Without necessarily taking sides on the empirical merits of these papers, our purpose is to provide a simple theoretical framework that might help in ordering the various definitions, and in providing some explanations for the seemingly confusing evidence. To do this, we follow Esteban and Ray (2001).² We first model the behaviour of players in case of conflict as a game and compute the equilibrium payoffs to all players. The status quo against which groups might rebel is characterized by a set of political institutions that channel the different opposing societal interests and turn them into a collective decision. Examples of such institutions are majoritarian or proportional democracies, dictatorships,

² This line is also adopted in Reynal-Querol (2002b).

oligarchies... We abstractly represent these institutions as alternative functions mapping the share of the population supporting each interest group into collective decisions. We take political institutions as given and hence disregard any potential endogeneity.³ Groups will rebel against the ruling political institutions whenever the outcome is worse than what they can obtain through conflict.

It is imperative to note that we distinguish between the intensity of conflict, *conditional on conflict breaking out*, and the likelihood that conflict actually occurs. The point that we make is very simple. When society is highly polarized, there may actually be a wider range of status-quo allocations that groups are willing to accept. This is because the potential cost of rebellion is extremely high, and this cost of conflict serves as the guarantor of peace. Put another way, if conflict is very costly as it will be in highly polarized societies, it is easier to find an agreement that is Pareto superior to the conflict regime. At the same time, if conflict were to occur for some reason, its intensity would be higher in polarized societies. It follows that the intensity of conflict (conditional on its occurrence) and the likelihood of conflict may move in opposite directions with respect to changing polarization.

In particular, when the cost of conflict is low, the parties will more easily reject proposals that slightly depart from what they can get through conflict. In the spirit of the fractionalization vs polarization controversy this argument can be summarized as follows. Highly fractionalized societies might be more prone to the onset of conflict, but its intensity will be moderate. In highly polarized societies, the occurrence of open conflict should be rare but its intensity very severe. In this paper we develop this argument and show that: (i) measures of fractionalization and polarization tend to run in opposite directions, (ii) the onset of conflict critically depends on the political system in place, (iii) the occurrence

³ The point was already made by Lipset and Rokkan (1967) that political systems might be endogenous, influenced by the particular social structure of the country. Why then societies fail to adapt their institutions to the change of the environment so as to always prevent domestic conflict? A number of arguments have been put forward by Powell (2004) –incomplete information–, Fearon (1995), Powell (2006) and Leventoglu and Slantchev (2006) –inability to credibly commit–, and Esteban and Ray (2006c) –empty core– to explain the break out of conflict. We shall not pursue this line of enquiry here and will take the political system as given.

of conflict and the intensity of conflict also tend to move in opposite directions, (iv) the relationship between polarization or fractionalization and conflict is non-monotonic and (v) the intensity of conflict depends positively on the degree of polarization.⁴

Our paper is organized as follows. We start by comparing the indices of fractionalization and polarization. Section 3 develops a simple model of conflict based on the general class studied in Esteban and Ray (1999). In order to present the ideas in the sharpest form, in Section 4 we start our study of the occurrence and intensity of conflict focusing on the case of two opposing groups only. This case permits a very neat understanding of the causes of intensity of conflict and the causes of its occurrence. However, as we shall see, in the case of two groups the notions of fractionalization and polarization are undistinguishable from each other. In section 5 we generalize the results to the case of an arbitrary number of groups and examine the different performance of the indices of polarization and fractionalization. Section 6 concludes.

2. Polarization and Fractionalization

We begin by defining the indices of fractionalization and polarization.

The index of fractionalization F is intended to capture the degree to which a society is split into distinct groups. The measure has been widely used in studies that attempt to link ethnolinguistic diversity to conflict, public goods provision, or growth (see, e.g., Collier and Hoeffler (1998), Fearon and Laitin (2003), Easterly and Levine (1997) and Alesina, Baqir and Easterly (1999)).

Let n_i be the share of the population belonging to group i , $i = 1, \dots, G$. The fractionalization index is defined as the probability that two randomly chosen individuals belong to different groups. The probability that an individual of group i is chosen is n_i . Hence that probability that if chosen she is matched with

⁴ This discussion can also shed some light on the controversy on the stabilizing or destabilizing effects of “polarity”; a classic the international relations literature. See Waltz (1964) and Deutsch and Singer (1964), for instance.

someone from another group is $n_i(1-n_i)$. It follows that the probability that any two individuals belong to different groups is

$$(1) \quad F = \sum_i n_i(1-n_i) = 1 - \sum_i n_i^2.$$

F is a strictly concave function of each population share. From this strict concavity we can derive the following properties of F .

(a) Any transfer of population from a group to a smaller one increases F ;

(b) For a given *number* of groups, G , F is maximized at the uniform population distribution over these groups;

(c) Over the set of uniform distributions F increases with the number of groups ; and

(d) The split of any group with population n into two new groups with n' and n'' , $n' + n'' = n$, increases F .

Polarization is conceptualized in Esteban and Ray (1994) as the sum of inter-personal “antagonisms”. Antagonism results from the interplay of the sense of group identification (group size) and the sense of alienation with respect to members of other groups (inter-group distance, b_{ij}).⁵ Esteban and Ray’s polarization measure⁶ P can be written as

$$(2) \quad P(\sigma, \mathbf{b}) = \sum_i \sum_{j \neq i} n_i^{1+\sigma} n_j b_{ij},$$

⁵ Alternative notions of polarization not based on the identity/alienation framework have been proposed by Wang and Tsui (2000), Reynal-Querol (2002c), and Zhang and Kanbur (2001). Another alternative and – considerably cruder - specification of polarization which also does not include a proxy for intra-group homogeneity in the absence of information is the concept of dominance that Collier (2001) introduced. It qualifies societies as “dominated” if the largest group contains between 45 and 90% of the overall population.

⁶ Esteban and Ray (1994) examine the main properties of this measure. The interested reader can also see Duclos, Esteban and Ray (2004) for a measure of polarization for continuous distributions.

where \mathbf{b} is the matrix of inter-group distances and σ is a positive parameter that captures the extent of group identification. Esteban and Ray (see also Duclos et al (2004)) derive restrictions on σ that bound it both above and below. The exact form of these restrictions is not particularly important here, though we record for use below that σ must be less than 2.

A situation of particular relevance is the special case in which individuals in each group feel equally alien towards all groups other than their own. That is, $b_{ij} = b_i$ for all $j \neq i$. In this case P reduces to

$$(3) \quad P(\sigma, \mathbf{b}) = \sum_i n_i^{1+\sigma} (1 - n_i) b_i.$$

Observe that if we set $\sigma = 1$ and $b_i = 1$ for all i we obtain the measure of polarization introduced by Reynal-Querol (2002c); a special case of (2). This specific measure of polarization was later used in Montalvo and Reynal-Querol (2005) to test the relationship between polarization and conflict.

It is also true that we can formally set $\sigma = 0$ in (3), as well as $b_i = 1$ for all i , to arrive at the measure of fractionalization (1). We emphasize that this is a formal and not a conceptual connection: for (3) to be a measure of polarization it is necessary that σ be strictly positive, and—depending on the exact characterization—perhaps more than that.⁷ Nevertheless, it is useful to record that

$$(4) \quad P(\mathbf{1}, \mathbf{1}) = RQ \text{ and } P(0, \mathbf{1}) = F,$$

where the entry $\mathbf{1}$ stands for the matrix of all 1's.

In order to simplify the computations, in this paper we shall work with the special class of polarization indices, $P(\mathbf{1}, \mathbf{1})$, that is

$$(5) \quad P \equiv P(\mathbf{1}, \mathbf{1}) = \sum_i n_i^2 (1 - n_i).$$

⁷ For instance, Duclos, Esteban and Ray (2004) argue that σ is at least 0.25.

In order to examine the properties of P we start by observing that P is the sum of the function $p(n) = n^2(1-n)$ evaluated at the different n_i . But now $p(\cdot)$ is convex or concave as $n <(>) 1/3$. Therefore, we have the following properties for P .

Properties of P

(a') A transfer of population from a group to a smaller one increases P if both groups are larger than $1/3$. If the two groups are smaller than $1/3$ the equalization of populations will bring P down;

(b') For any given number of groups, P is maximized when the population is concentrated on two equally sized groups only;

(c') Over the set of uniform distributions P decreases with the number of groups, provided that there are at least two groups to begin with; and

(d') The split of a group with population n into two groups with n' and n'' , $n' + n'' = n$, increases P if and only if $n \geq 2/3$.

The contrast between the two sets of properties clearly shows that the two measures behave quite differently from each other, except when there are just two groups. The difference is clear: fractionalization is maximal when each individual is different from the rest while polarization is maximal when there are only two types of individuals.

The reader is referred to Montalvo and Reynal-Querol (2005) for further discussion on the difference between these two measures.

This completes our discussion of the indices. We now turn to a model of conflict and peace.

3. A Model of *War and Peace*

3.1. Conflict

We start with *conflict* as our first building block. In modeling conflict we follow Esteban and Ray (1999). We concentrate on a special case studied in that

paper: the class of conflict games called *contests*. Assume that there are G alternatives, $i = 1, \dots, G$. Individuals differ in the alternative they like the most and are indifferent over the other available alternatives. Individuals in a specific group i are all alike, in that they like alternative i the best, and the difference in valuation between their most preferred alternative and any other is the common value b_i . Let n_i denote the relative size of group i . Note that the alternatives here are *public goods* because their valuation by the individuals is independent of the number of beneficiaries. Therefore, we can think of alternatives as different kinds of public goods to be financed by the public budget.

By a *political system* we shall refer to a particular way of choosing among the different alternatives. By *conflict* we mean a challenge to such a system, which is costly. Specifically, we take the following view. Conflict entails resource contributions r_i (to be determined presently) from every member of group i , so that the overall contribution of group i is $n_i r_i$. In the absence of a political rule, the particular alternative that will eventually come about is seen by the players as probabilistic. The probability that alternative i will be established is assumed to be equal to the resources $n_i r_i$ expended by group relative to the total resources R expended. In short, the probability of success p_i is just

$$(6) \quad p_i = \frac{n_i r_i}{\sum_j n_j r_j} \equiv \frac{n_i r_i}{R},$$

where R is the sum of all the group contributions. In the sequel, we shall take this very R to be a measure of the overall intensity of conflict (or wastage) in the society.

To understand how contributions are determined, suppose that there is a utility cost of spending r_i ; call it $c(r_i)$. Take this function to be of the constant-elasticity form

$$(7) \quad c(r_i) = \frac{r_i^{1+\alpha}}{1+\alpha}, \text{ with } \alpha > 0.$$

Given the resources expended by the others, r_{-i} , the expected utility of an individual of group i when spending r_i is

$$(8) \quad E(u_i(r_i)) = p_i b_i - c(r_i) = \frac{n_i r_i}{\sum_j n_j r_j} b_i - \frac{r_i^{1+\alpha}}{1+\alpha}.$$

Expected utility is clearly concave in r_i and hence the utility maximizing level of expenditure can be characterized by the first order condition:

$$(9) \quad \frac{n_i}{\sum_j n_j r_j} \left(1 - \frac{n_i r_i}{\sum_j n_j r_j} \right) b_i = \frac{n_i}{R} (1 - p_i) b_i = r_i^\alpha.$$

An *equilibrium* of the conflict game is a vector \mathbf{r} such that (9) is satisfied for all $i=1, \dots, G$.

There is always an equilibrium of the conflict game. Esteban and Ray (1999) demonstrate, furthermore, that if $\alpha \geq 1$ then such an equilibrium must be unique.

In order to simplify the computations we shall focus on the case of symmetric valuations, with $b_i = 1$ for all i , and $\alpha = 1$.

Multiplying both sides of (9) by $\frac{n_i}{R}$ we obtain

$$(10) \quad \left(\frac{n_i}{R} \right)^2 (1 - p_i) = p_i,$$

And transposing terms, we conclude that

$$(11) \quad p_i = \frac{n_i^2}{n_i^2 + R^2}.$$

The equilibrium value of R has to be such that the sum of the probabilities given by (11) adds up to unity, that is

$$(12) \quad \sum_i p_i = \sum_i \frac{n_i^2}{n_i^2 + R^2} = 1.$$

The LHS of (12) is strictly decreasing in R . Using (10), it is immediate that when R goes to zero the LHS tends to $G > 1$ and that when R tends to infinity the LHS tends to zero. Therefore a solution to (12) always exists and it is unique.

Substituting the equilibrium R into (11) yields the equilibrium probabilities.

In order to obtain a useful expression for equilibrium payoffs we multiply both sides of (9) by $\frac{r_i}{2}$ to see that

$$(13) \quad \frac{1}{2} p_i (1 - p_i) = \frac{1}{2} r_i^2 \equiv c(r_i).$$

Using (13) in (8) yields

$$(14) \quad E(u_i(r_i)) = \frac{p_i(p_i + 1)}{2}.$$

For the case of two groups, $G = 2$, setting $n_1 = n$ and $p_1 = p$, the equilibrium values are easy to compute:

$$(15) \quad p = n,$$

$$(16) \quad R = \sqrt{n(1-n)}, \text{ and}$$

$$(17) \quad E(u_1(r_1)) = \frac{n(n+1)}{2} \text{ and } E(u_2(r_2)) = \frac{(1-n)(2-n)}{2}.$$

The equilibrium payoffs to conflict for player i will be simply denoted u_i .

3.2. Peace

In the previous subsection we have examined the equilibrium outcome of a conflict game *under the assumption that conflict actually takes place*. Now we

are in a position to compare a conflictual situation with that of peace. Under peace, individuals must accept the payoff that the ruling political system allocates to them. As mentioned before, the political allocation can be interpreted as the share of the public budget allocated to the production of the type of public good most preferred by each group. We define a *policy* to be a vector γ of shares, with γ_i denoting the share of group i . Hence, we can interpret γ as a “compromise policy” composed of a convex linear combination over the available alternative types of public goods.

Formally, we shall have *peace* whenever

$$(18) \quad u_i \leq \gamma_i \text{ for all } i = 1, \dots, G.$$

It is trivial but nevertheless useful to observe that whether we have conflict or peace critically depends on what the ruling political system delivers to the different contending groups.

A political system is a particular way of mapping the population shares supporting each alternative into policies.

We shall examine here various political systems and check for their ability to guarantee peace. Specifically, we shall study dictatorship, fixed shares, majoritarian rule and proportional rule.

Our first example of a political system is a *dictatorship*. This will be the case when the alternative preferred by some group i is brought into effect, irrespective of the number of individuals for whom this is the best choice. If group i is the dictator, then $\gamma_i = 1$ and $\gamma_j = 0$ for all $j \neq i$.

The second case is *fixed shares*, which generalizes a dictatorship. The policy consists of a vector γ assigning a share to each group independent of its population size. There are many instances of such a political system. Various political bodies have fixed proportional representations of the different opposing interests (often rural vs urban). There are also cases where the chairs of the

two chambers have to alternate between the different ethnic or religious groups in the country.⁸

The *majoritarian rule* generates the policies that earn the support of a majority of citizens. For the case of $G = 2$ this is very easy to define: $\gamma_i = 1$ if and only if $n_i > 1/2$.⁹ For $G > 2$ the characterization of the policies resulting from a majoritarian rule is more intricate as it involves the formation of a majoritarian coalition. In some special environments there is a well-defined pivotal group (the *median* voter) who can impose its preferred policy to the rest of the majoritarian coalition. This is not the case here and hence most of what we can say will be restricted to the two-group case.

Finally, the *proportional rule* produces the policy that assigns to each group a share equal to its population size: $\gamma_i = n_i$. Parliamentary representations satisfy this rule for most countries (not in the UK where each seat corresponds to one constituency). Although most decisions simply require a majority vote in the chamber, the resulting policies tend to give some weight to the minoritarian opposition. Multi-level government also contributes to give to the different groups an overall weight that brings them closer to their population share.

In the next section we study the relationship between polarization, fractionalization and conflict under the different political systems for the case of two groups. In section 5 we generalize to G groups.

4. Polarization, fractionalization, conflict and the political system ($G=2$)

⁸ This was the first constitutional arrangement for the Lebanon after independence. The constitution established that the president had to be a Christian. The faster population growth rate among the Muslim population made this provision untenable and possibly contributed to the outbreak of the civil war. Another example is the EU “rotating presidency” across the member countries with a frequency that is independent of their population.

⁹ This is an extremely stylized representation of the majoritarian rule. Real world majoritarian democracies do not work like this. A number of written and/or unwritten rules protect minorities from the tyranny of the majority. This observation has led Lijphart (1977) to launch the concept of “consociational” policies that end up producing an outcome that approaches the proportionality rule. Lijphart has been a steady supporter of “consociational” constitutions for countries with deep ethnic cleavages.

We are interested here in two quite different aspects of conflict. In the first place, we want to characterize the relationship between the intensity conflict and polarization *when conflict actually takes place*. This relationship is independent of the political system. Secondly, we wish to identify the relationship between polarization and the occurrence of conflict (or peace!).

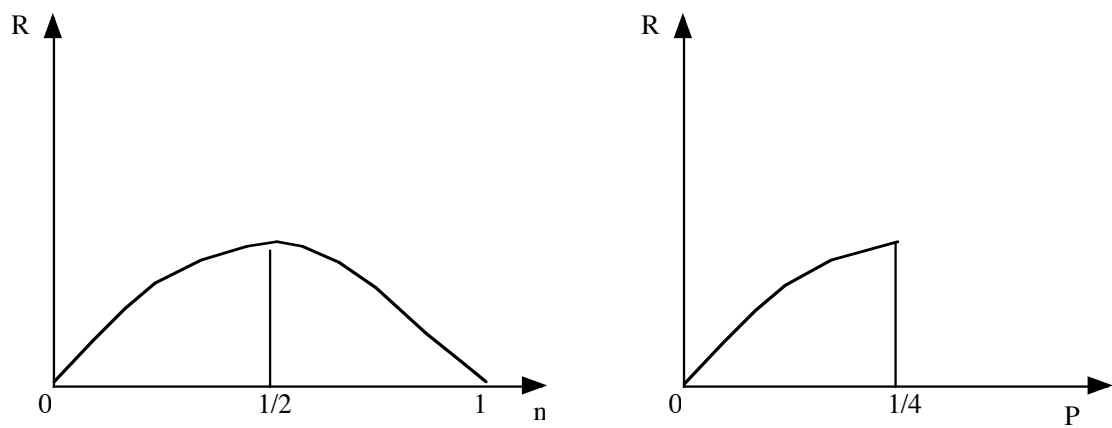
Intensity of conflict

We start by noting that for $G = 2$ the measures F and P (and hence RQ) are proportional to each other. Furthermore, they all attain their maximum at $n = 1/2$. It follows that any comparative test of the performance of P (or RQ) relative to F as a predictor of conflict should focus on cases with $G \geq 3$. We discuss this case in the next section.

In view of (16), the level of conflict R is the *square* root of P and hence conflict intensity is an increasing function of polarization and of fractionalization.¹⁰

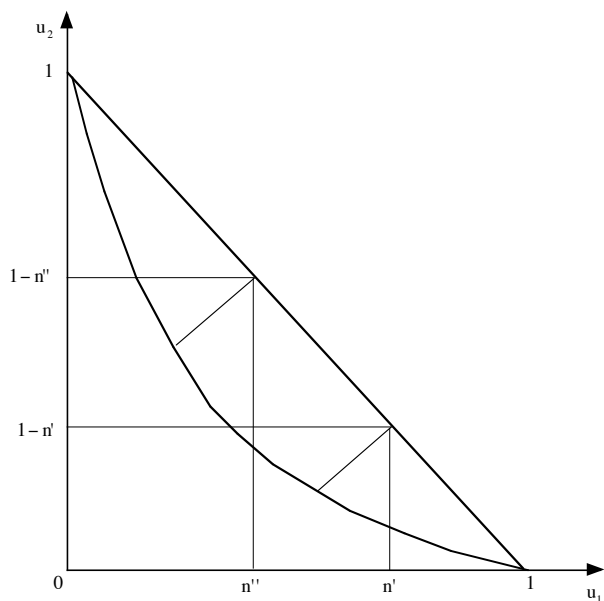
Figure 1

¹⁰ If we drop now the restriction that $a = 1$ but retain that $b = 1$, $P(a,1)$ ceases to be proportional to F , but continues to behave like it. Indeed, $P(a,1)$ is concave and attains its maximum at $n = 1/2$. Therefore, we will still have that increases in $P(a,1)$ go with increases in the level of conflict R . Things are different when we allow for asymmetric inter-group distances. It can be readily verified that if $b < 1$ ($>$) both polarization and conflict are maximized at (two different) values $n^P, n^R > 1/2$ ($<$). Therefore, except for values of n within this interval, the level of conflict will be strictly increasing with polarization. The non-monotonicity with respect to F and RQ will be for n in the interval $(1/2, n^R)$.



In Figure 1 we plot the intensity of conflict as a function of the population shares n (left figure) and of the corresponding level of polarization P (right figure). Intensity is maximal for $n = \frac{1}{2}$, that is when polarization is also maximal with $P = \frac{1}{4}$.

Figure 2



It will also be useful to record the equilibrium utility payoffs as given by (17) and which we represent in Figure 2. These payoffs depend on the population distribution parameter n . The equilibrium utility for each player is the win probability $p = n$ minus the cost of the resources expended in conflict, equal for both types of players for $G = 2$. The win probabilities are points on the straight line between (0,1) and (1,0), the utility possibility frontier. Given n , from the corresponding point on the frontier we move inwards along a 45° line for a

length equivalent to the utility loss caused by the spent resources. This gives us a utility equilibrium pair. As we vary n we generate all the points of the equilibrium payoff curve. The maximum distance between the payoff curve and the frontier is at $n = \frac{1}{2}$ where the conflict loss is maximal.

So much for intensity. Let us now turn to the occurrence of conflict. This depends on the payoffs obtained in peace and these in turn depend on the political system.

Dictatorship

The first political system we examine is dictatorship. Will there ever be peace? The answer is no. The reason is simple. In equilibrium conflict, all players receive a strictly positive payoff because they could have opted for contributing nothing to conflict, thus guaranteeing for themselves a payoff of zero. Hence, for a non-dictator obtaining a peace payoff of zero is always dominated by the conflict payoff.

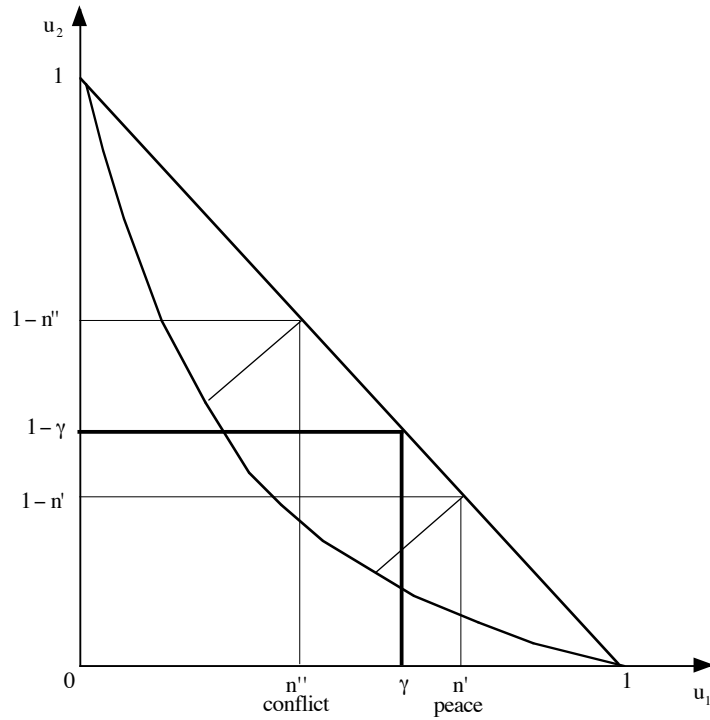
Fixed shares

We next examine the case of fixed shares γ .

The necessary and sufficient condition for conflict is that either

$$(19) \quad \frac{n(1+n)}{2} > \gamma \quad \text{or} \quad \frac{(1-n)(2-n)}{2} > 1-\gamma.$$

Figure 3



The situation is captured in Figure 3. Consider the peace share γ and the corresponding utility payoff. For a population parameter like n' the payoffs to conflict are dominated by the peace payoff for the two players. However, if we decrease sufficiently the population share of the first group—all the way down to n'' , the second group would have a strong advantage over the first in conflict and thus prefers conflict to the peace payoff.

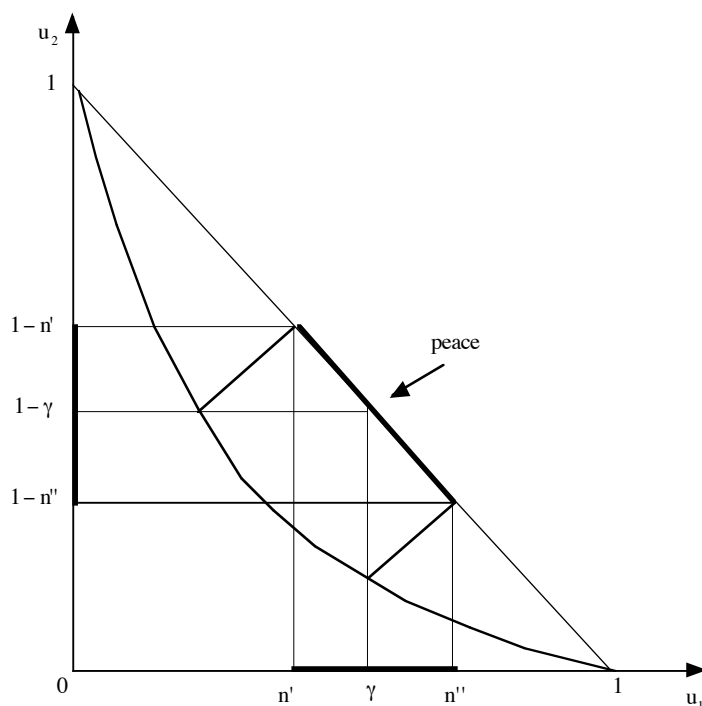
To be more specific, let us rewrite the inequalities in (19) as

$$(20) \quad n - \frac{n(1-n)}{2} > \gamma \text{ or } n + \frac{n(1-n)}{2} < \gamma.$$

The LHS of the two inequalities is strictly increasing in n (one convex and the other concave). Therefore, there exist n' and n'' such that if $n \in [n', n'']$ there is peace and conflict otherwise.

In Figure 4 we depict the values of n for which, given γ , we shall have peace. These are the values of n corresponding to the points on the equilibrium utility curve at which one of the two players is indifferent with respect to the peace payoff.

Figure 4



Clearly, the interval of values of n for which there will be peace depends on the bias exhibited by the fixed-shares policy γ . Let us take as a benchmark the case of equal treatment of the two groups of players with $\gamma = \frac{1}{2}$. From our previous analysis it follows that for very low polarization (i.e. for very low or very large n) there will be conflict, but its level will be low. As polarization increases the level of conflict will increase too. But, further increases in polarization will produce peace and bring the level of conflict down to zero. *The overall relationship between polarization or fractionalization and conflict is therefore non-monotonic.*

We can address the complementary question of the range of policies γ that would guarantee peace for given n . This range is given by the gap between the two bounds: $n(1-n)$. Hence the widest range for peaceful policies corresponds to $n = 1/2$. *High polarization allows for a wider choice of peaceful fixed-share policies.* The intuition for this result is straightforward. If there is conflict, higher polarization produces larger losses. Hence, it is only when the policy is very biased against one group that that group will decide to incur the heavy cost of conflict. With low polarization the costs are smaller and hence a lower bias in γ might be enough to trigger conflict.

Majority rule

The case of majority rule is equivalent to letting the largest group become a dictator. By the same argument as before, we shall never have peace as the minoritarian group will always obtain a higher payoff under conflict than under peace. Hence, *with majority rule we shall always have conflict¹¹ and the level of conflict will positively depend on the degree of polarization.*

Proportional rule

We start by noting that in the previous case of fix shares, in view of (20), when γ is sufficiently close to the win probability of that group peace will not be challenged. Under our assumptions, $p = n$ and hence making $\gamma = n$ would guarantee peace. This precisely is the proportional rule that gives each group a share equal to its population size, that is, $\gamma_i = n_i$.

Therefore, for *symmetric valuations we should never observe conflict under the proportionality rule.*¹²

The intuition for this result is that the proportionality rule gives to each group a weight that is close to their win probability under conflict. Hence, it never pays to challenge the peace allocation.¹³ As we will see, this result is specific to the two-group case and does not extend to the case of a larger number of groups.

¹¹ Let us insist in that this statement is not meant to be empirically relevant as none of the existing majoritarian democracies permits the tyranny of the majority. From an empirical point of view the practical distinction between majoritarian and proportional democracies is far from clear. The use of the notion of “inclusiveness” of a political system as in Reynal-Querol (2005) might be more relevant.

¹² This is no longer true for asymmetric valuations. The equilibrium p can be made arbitrarily close to unity by choosing b for one group sufficiently close to zero.

¹³ This result seems to substantiate Lijphart’s view that “consociational” systems, because they are essentially proportional, permit peaceful arrangement in ethnically divided societies. [see more in footnote 13]

Diagrammatically, we can see in Figure 2 that the point $(n, 1-n)$ always dominates the conflict equilibrium payoffs.

Summing up

In this section we have obtained two main results. The first one is that it does matter for conflict which political system is in place. Dictatorships and majoritarian systems never yield peace. Fixed shares may give peaceful outcomes for some parameter values. The proportional system always yields peace, at least whenever there are just two groups.

The second result is that while the *intensity* of conflict is positively related to the degree of polarization, the *incidence* of conflict is not. Only in the fixed shares system does the incidence of conflict depend on the distribution of the population across the two groups. For the other three political systems the incidence of conflict is independent of the distribution (and hence of the degree of polarization). For the fixed shares system conflict is more likely at low levels of polarization and peace more likely at high levels. Therefore, if there is any relation between conflict and polarization this is non-monotonic.

Our analysis also suggests that if one wishes to test for the occurrence of conflict, the political system appears to be a key variable, along with the degree of social polarization (or fractionalization).

5. Polarization, conflict and political rules with several groups

We shall examine now whether our previous conclusions can be extended to the general case of several groups.

Intensity of conflict

We start with the relationship between polarization and the intensity of conflict, R . The relationship between the intensity of conflict and polarization has been extensively studied in Esteban and Ray (1999, section 6). Using Esteban and Ray (1999, expression 16) we can write

$$(21) \quad R = \sum_i \frac{n_i}{p_i} n_i^2 (1 - n_i) b_i.$$

Comparing (21) with the measure of polarization P in (4) we can observe that, if $p_i = n_i$, the level of conflict R would be equal to the index of polarization $P(1, \mathbf{b})$.

The n/p ratio is determined in equilibrium and will generally be different from unity. Therefore, how closely related P is to R critically depends on how much n/p varies across the different groups in equilibrium. It can be shown that the case in which $n_i/p_i = 1$ for all i is specific to the symmetric case for $G = 2$ or for uniform distributions over $G > 2$ groups. Therefore, on these grounds alone, we should *a priori* expect a positive but incomplete association between polarization $P(1, \mathbf{b})$ [and hence $P(1, 1) = RQ$] and the level of conflict R . However, the discussion that now follows suggests that there are pretty tight connections between the two.

Drawing on the results in Esteban and Ray (1999) we can restate the following properties of R , implicitly determined in (12), to be contrasted with the properties of F and P presented in section 2.

(i) A transfer of population from a group to a smaller one increases R if both groups are larger than $1/3$. If the two groups are small enough the equalization of populations will bring R down;

(ii) For any given G , R is maximized when the population is concentrated on two equally sized groups only;

(iii) Over the set of uniform distributions R decreases with the number of groups G ; and

(iv) The split of a group with population n into two groups with n' and n'' , $n' + n'' = n$, increases R if and only if the group size is sufficiently large. If n is small, the split will decrease R .

Do the properties of our theoretical model align with our intuition on the intensity of conflict? Consider conflict among three groups of varying size. Property (i) says that equalizing the size of the two largest groups will increase conflict while reducing the size of the second largest group at the benefit of the smallest will reduce conflict. Property (ii) appears to conform to the common intuition that conflict is worst when society is split into two equally sized groups. In the case considered by Property (iii) each group becomes progressively smaller, while its collective opponent (the rest of the groups) becomes larger. In this case the smaller groups will commit less resources into conflict. As for Property (iv), consider first the case of a monolithic society that gets split into two distinct groups. This must increase the intensity of conflict. The same has to be true even if the initial society was not monolithic, but had a small “dissident” group. But suppose now that after the first split the second sized group splits into two smaller groups. Then we would expect that conflict would come down because now the untouched group has become relatively larger than the others. The smaller groups may not be willing to contribute a lot of resources to conflict.

In sum, the properties displayed by our conflict model do not seem to contradict our intuitions about conflict intensity.

Let us now compare the properties of R and P . It is immediate that the two sets of properties describe movements in the same direction for the type of population changes considered. Hence, we should expect a strong positive relation between polarization and conflict intensity. [See a parametric illustration below]

How does the index of fractionalization F behave relative to R ? Property (i) of R is not satisfied by F . Property (a) of F says that any equalization of sizes will increase F . In contrast, R may go either up or down depending on the size of the groups involved. Properties (ii) and (b) are aligned as long as there are two groups in conflict to start with. With more groups F is maximized at the uniform

distribution while R continues to be maximal when the population is concentrated on two equally sized groups. Properties (iii) and (c) are exactly the opposite of each other. Finally, when we compare Properties (iv) and (d) we observe that any split always increases F , while R may either decrease or increase depending on the size of the broken group.

We can thus conclude that we can expect a strong positive relationship between polarization and conflict, and a weak and (if anything) negative relationship between fractionalization and conflict, at least insofar as intensity is concerned.

We now turn to an analysis of the incidence of conflict when there are more than two groups.

Dictatorship and majoritarian rule

Notice our arguments on the impossibility of peace under dictatorship or the majoritarian rule did not depend on the number of groups. In both cases, the excluded groups will obtain a lower payoff than what they can obtain under conflict.

Fixed shares

From (14) we have that there will be conflict whenever

$$(22) \quad \frac{p_i(1+p_i)}{2} > \gamma_i \text{ for some } i = 1, \dots, G.$$

Using (11) in (22) we obtain that the condition for conflict is

$$(23) \quad u_i = \frac{1}{2} \frac{n_i^2}{n_i^2 + R^2} \left(1 + \frac{n_i^2}{n_i^2 + R^2} \right) > \gamma_i.$$

Consider any given vector of shares γ and a particular group of size n_i . Observe that the payoff u_i can take values in $(0,1)$ depending on R . Therefore, the condition for conflict is most likely to be satisfied when R is small and hence polarization is small too. To be precise, suppose that all the remaining groups have the same size, $n_j = \frac{1-n_i}{G-1}, j \neq i$. It can be readily verified from (12) that R is strictly decreasing in G . It follows that there is a G sufficiently large so that a uniform distribution over the $G-1$ remaining groups would induce group i to prefer conflict. Note that as G becomes large polarization comes down and fractionalization goes up. Therefore we shall see conflict with low levels of polarization and high levels of fractionalization, but the intensity of conflict will be low.

In the discussion above, observe that it is the untouched group, the group which has become larger relative to the others, is the one who prefers conflict to peace. Hence, even in this case, one might argue that it is not high fractionalization as such that precipitates conflict but the *coexistence* of one large group with numerous small groups. In fact, if we now equalize the size of all the groups, thus increasing F and decreasing P , no group would have an incentive to challenge the peace share and we would have peace with higher fractionalization.

To sum up, for the egalitarian fixed shares policy, *conflict will not occur in economies with high polarization/low fractionalization*. For distributions displaying low polarization/high fractionalization, the relation between conflict and F or P will be non-linear. *Conflict will be most likely for distributions with one large group and many small ones* (and hence with relatively high fractionalization and low polarization).

As the rule of fixed shares departs from egalitarianism, the occurrence of conflict will critically depend upon the bias introduced by the rule.

Proportional rule

Once again, from (14) we have that under the proportional rule there will be conflict whenever

$$(24) \quad \frac{p_i + 1}{2} > \frac{n_i}{p_i} \text{ for some } i = 1, \dots, G.$$

In the previous section we have seen that for $G = 2$ the proportional rule always guarantees peace. Does this property extend to $G > 2$?

A first observation is that for the distributions under which the equilibrium win probabilities are very close to the population shares condition (24) will not be satisfied and we shall observe peace. We shall only have conflict when p_i is sufficiently larger than n_i for some group i .

Using (10) in (14), we can rewrite condition (24) as

$$(25) \quad \frac{1}{2} \frac{n_i}{n_i^2 + R^2} \left(1 + \frac{n_i^2}{n_i^2 + R^2} \right) > 1.$$

The LHS of (25) can take values in $(0, \frac{1}{2n_i})$, depending on R . Provided $2n_i < 1$, we have already seen that there is a distribution of the population (for G sufficiently large) so that group i will prefer conflict over peace. Esteban and Ray (2001) demonstrate that under these assumptions there always are distributions for which (25) is satisfied for one group. Here are two numerical examples: $G=5$ with one group being 1/3 of the population and the other four of size 1/6; and $G=4$ with one group of size 1/2 and the other three of size 1/6.¹⁴

As in the case of fixed coefficients conflict occurs in very skewed distributions by size. One large group together with a number of small sized groups is the type of distribution that would be more likely to generate open conflict. Because

¹⁴ The case of India has been taken as a critical test for Lijphart's claim that "consociational" systems—hence proportional—are guarantors of ethnic peace. Most of the debate, Lijphart (1996) and Wilkinson (2000), has focused on whether India was more "consociational" under Nerhu or more recently. Our analysis suggests that the change in population sizes of Hindus, Muslims and others that has actually taken place in India in 1961-2001 might also have a role in explaining the evolution of ethnic conflict.

of the returns to scale in conflict, the win probability of the large group may amply exceed its population share. Furthermore, precisely because of the returns to scale the small groups will be deterred from expending much resources in such an uneven conflict and hence we shall observe a low conflict loss R . High win probabilities together with small aggregate losses make conflict preferable to peace as far as the large group is concerned.

Therefore, we may conclude that with $G > 2$ *under the proportional rule we may have conflict*. This will be associated with distributions with low polarization and high fractionalization. However, the relationship will be non-monotonic: additional increases in fractionalization may bring peace rather than further conflict.

In order to illustrate this relationship consider the following parametric example. There are three groups with $n_1 = 1/2$, $n_2 = \lambda 1/2$, and $n_3 = (1-\lambda) 1/2$, $0 \leq \lambda \leq 1/2$. When $\lambda = 0$ we have two groups with the same population and thus maximal polarization. When $\lambda = 1/2$ we shall have the same first group facing two groups of half the size. F and P can be computed to be

$$(26) \quad F(\lambda) = \frac{1 + \lambda - \lambda^2}{2}, \text{ and}$$

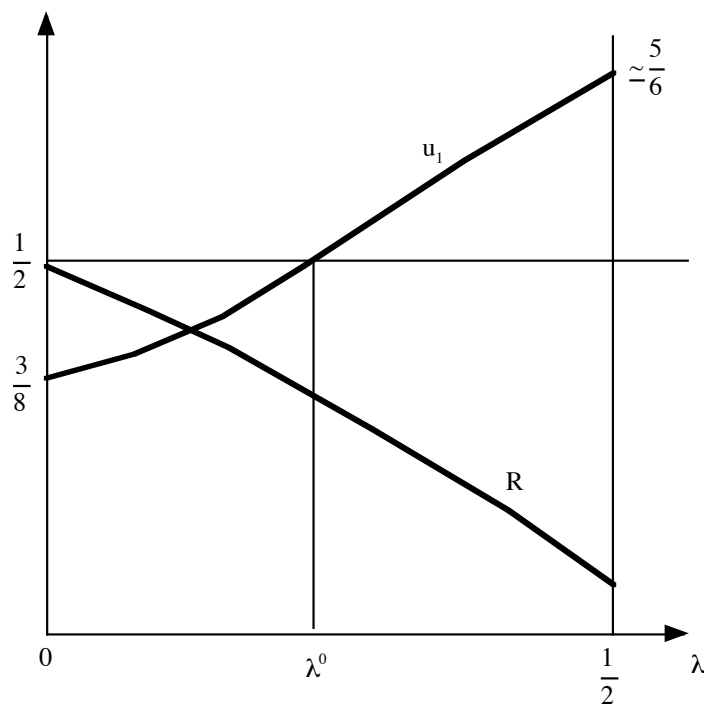
$$(27) \quad P(\lambda) = \frac{3 - \lambda + \lambda^2}{8} = \frac{1}{2} - \frac{F}{4}.$$

From (27) it is plain that when λ changes fractionalization and polarization move in opposite directions: as we move away from the perfect bipolar distribution P comes down but F goes up.

Using this parametrization for the distribution of the population in expression (12) we implicitly obtain the equilibrium intensity of conflict R as a function of λ . Totally differentiating, we obtain that R decreases as λ increases, i.e. as P decreases and as F increases. Conflict intensity goes from $R(0) = 0.5$ to $R(1/2) = 0.211$. This is depicted in Figure 5.

Whether there will be conflict or peace under the proportional rule depends on whether the untouched group –always with population $\frac{1}{2}$ -- obtains a conflict equilibrium utility higher or lower than $\frac{1}{2}$. In Figure 5 we also depict u_1 as a function of λ . Not surprisingly, as λ increases group 1 is facing smaller and smaller enemies. Hence, u_1 increases with λ . The large group obtains a higher utility from conflict the less polarized the distribution is. The equilibrium utility goes from $u_1(0) = 0.375$ to $u_1(1/2) = 0.837$. It follows that for low λ the equilibrium utility of group 1 will be below the peace payoff and there will be peace. This corresponds to the highest levels of polarization and lowest of fractionalization. For $\lambda > \lambda^\circ$ [see Figure 5] there will be conflict. Therefore, open conflict will be associated with low polarization and high fractionalization.

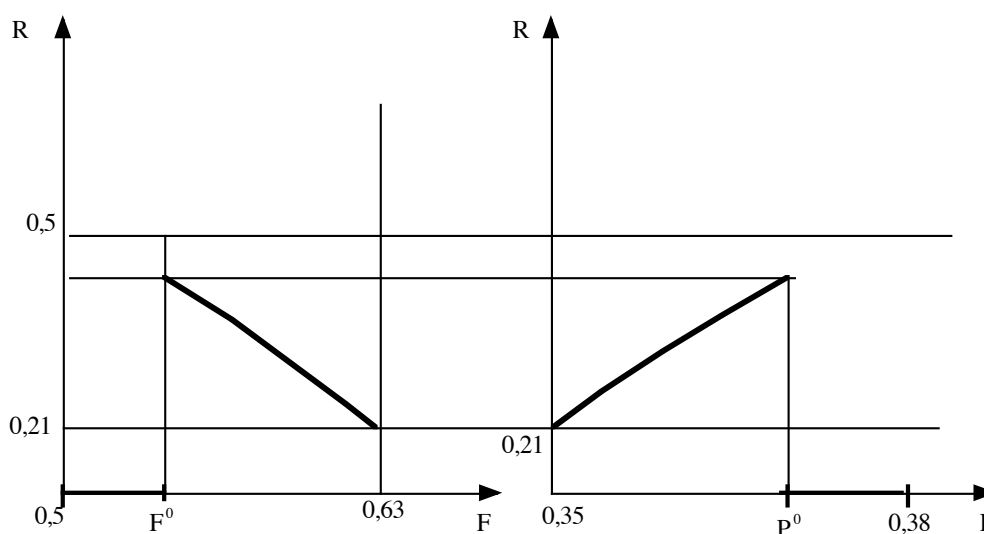
Figure 5



We finally combine the intensity with the occurrence of conflict and derive the relationship between observable intensity of conflict and both fractionalization and polarization. This is depicted in Figure 6. As we can see, in both cases the relation is nonmonotonic. For the case of fractionalization, there is peace until the threshold level F° is reached. At this point, there is conflict and it attains its maximum intensity. For higher values of F we continue to have conflict but its

intensity monotonically comes down. The relationship between P and observable intensity of conflict is the other side of the coin. Open conflict occurs at low levels of polarization. As polarization goes up the intensity of conflict raises until the threshold P^0 is attained. For higher levels of polarization the costs of conflict are so high that we will observe peace. The two functions are depicted in Figure 6.

Figure 6



Summing up

When we consider distributions with more than two groups it is still true that the occurrence of conflict critically depends on the particular political system in place. Dictatorship and the majoritarian rule can never bring peace, as we already observed for $G = 2$. But in general, both fixed shares and proportional rule fail to universally guarantee peaceful outcomes. We shall not see conflict neither for very low nor for very high levels of fractionalization¹⁵ and a similar (but inverse) pattern would be followed by polarization.

¹⁵ This seems to contradict the result obtained by our parametric example above. This is due to the very special change in the distribution that our parametrization allows for. Consider for instance our limit case with $l = \frac{1}{2}$ ($n_1 = \frac{1}{2}$, $n_2 = \frac{1}{4}$, $n_3 = \frac{1}{4}$). Fractionalization is maximal and we still have conflict. However, if we now move to $n_1 = n_2 = n_3 = \frac{1}{3}$ —not allowed by our parametrization—fractionalization would be even higher but there would be no conflict.

Concerning the general relationship between polarization, fractionalization and conflict our results suggest that they will be significantly nonlinear. Under some political systems the occurrence of conflict is independent of the shape of the distribution while in other systems it does depend on the shape. Under the first class of political systems the intensity of conflict will be closely (positively) related to the degree of polarization (and negatively to fractionalization). Under the second class (fixed and proportional shares) we shall observe zero intensity at high and very low levels of polarization (and fractionalization). For the range of levels of polarization for which we shall have conflict, higher polarization will be positively related to higher intensity of conflict. As far as fractionalization is concerned there seems to be no regular relationship between its level and the intensity of conflict.

All these results suggest that there may be more to be learned from empirical exercises that put all the evidence together and also attempt to control for the political system of each country.¹⁶

6. Conclusions

We provide an analytical framework that permits an interpretation of recent empirical exercises attempting to identify a meaningful relationship between population distributions over opposing groups and emergence or intensity of conflict. Conflict breaks out when the payoffs delivered by the political system fall short of what one group can obtain by precipitating conflict. While the intensity of conflict clearly depends on the shape of the distribution, the occurrence of conflict also depends on the responsiveness of each political system to the popular support for each of the competing alternatives. When we

¹⁶ Political scientists have been aware for long of the critical role played by the political institutions in preventing domestic conflict. The work of Lijphart (1977) is fundamental here. See also the recent controversy between Horowitz (2006) and Fraenkel and Grofman (2006) on the effectiveness of constitutional *engineering*. Our point is that in spite of this important line of literature, empirical tests on the determinants of conflict have very seldom controlled for the political system. An exception is the work by Reynal-Querol (2002b, 2005) and by Schneider and Wiesehomeier (2006b) who do study the relationship between political systems and domestic conflict.

combine occurrence with intensity, the relationship between conflict and polarization/fractionalization becomes significantly non-linear and contingent on the ruling political system.

The rationale behind our result is quite straightforward. Conflict is costly. That is overall payoffs are less than what are achievable under peace. The costlier conflict is (if it actually takes place) the easier it becomes to assign payoffs to groups that Pareto dominate what they can obtain under conflict. Therefore, only the political systems with very unfair outcomes (such as dictatorship or majoritarian rule) will be always be challenged even when the cost of doing so is high. However, under “fairer” systems no group would be willing to pay too high a cost to obtain a different payoff. Therefore, it is only when conflict is nearly costless to one group (such as the case of one large group and a number of small opponents) that the outcome of the political system will be challenged, by that precisely that large group.¹⁷

Highly polarized situations may well be fairly peaceful. This is what happened during the Cold War period, for instance. The cost of challenging the international status quo was so immense that even if one of the two sides considered the division of international power disproportionate it could not—or would not—trigger a world conflict. At the same time, when polarization is extremely low, there is little to fight about. Consequently, we would expect the overall degree of conflict to be maximal in societies with intermediate levels of polarization.

What, then, are the hopes for the empirical exercises that try to identify a relationship between polarization/fractionalization and conflict? Two recommendations appear to emerge. First, there should be a serious attempt to account for the nonlinearity. For instance, in a parametric context, some progress may be possible by entering both polarization and its square on the

¹⁷ Note the similarity of this point with the findings of Collier (2001) on the dominant ethnicity provoking civil war. One should qualify these points, however, by observing that small groups can provoke conflict when private goods are at stake. For more on this issue, see Esteban and Ray (2006b).

right-hand-side of a regression.¹⁸ But the prescription is simply this: the empirical specification needs to be more firmly grounded in theory, even if that theory is extremely simple.

Second, we have seen that the incidence of conflict depends not only on the shape of the distribution but also critically on the ruling political system. Alternative political systems perform quite differently in guaranteeing peace. For the countries with political systems that always yield conflict we shall observe that the intensity of conflict is (roughly) positively related to polarization (and negatively) to fractionalization. However, in countries with political systems that may yield peace, the occurrence and intensity of conflict will typically have a highly non-linear relationship with polarization and/or fractionalization. It follows that the exercise critically demands that political systems be controlled for.¹⁹

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¹⁸ On a similar issue arising in the empirical debate on inequality and growth (though for very different reasons), see Banerjee and Duflo (2003).

¹⁹ Reynal-Querol (2002b) has tested the relationship between type of political system (their degree of *inclusiveness*) and conflict. However, in Montalvo and Reynal-Querol (2005) the empirical test of whether polarization or fractionalization are the best predictors for conflict does not use the political systems as controls. Schneider and Wieserhomeier (2006b) also emphasize the critical role played by the political institutions in the occurrence of conflict.

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